Automatic Loop Shaping (ALS) - A PID Tuning Method for Systems with Real Left Half Plane Poles

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Abstract

We present a new PID controller tuning method for higher order process models. Existing methods, such as the Ziegler-Nichols (ZN) approach in time domain, are not designed according to stability measures, an essential property in feedback control. Furthermore they deliver one set of parameters for the controller without regard of the special performance requirements. In order to guarantee stability and certain robustness to model uncertainties and modeling errors in practical applications we need a criterion e.g. such as phase margin, which is applied in the frequency domain. The existing methods in the frequency domain propose a tuning algorithm where the phase margin is often used only to determine the controller gain [1, 2, 3]. The other parameters are sometimes tuned by canceling the poles of the plant [1]. Depending on the required time domain features, even better results can be obtained if all controller parameters are fine tuned by loop shaping. Loop shaping is a trial and error procedure in frequency domain whereby the parameters of the controller are set in order to balance many different requirements [4]. This method is often time consuming, especially if a predetermined phase margin and different performance specifications are requested. Therefore we propose a supportive loop shaping approach which is based on mathematical derivation in the frequency domain. Our method guarantees a chosen phase margin and a fast controller response. Furthermore the user is enabled to influence different features according to given performance requirements.

We apply our approach to a simulated fill level plant and show its performance, comparing it with a Ziegler-Nichols method tuned PID controller and a Bode diagram design approach based on pole cancellation [1].

1 Introduction

The proportional-integral-derivative (PID) controller is by far the most common controller used in practical application [1]. More than 90% of technical control problems can be solved with a PID controller, furthermore the implementation of PID controllers in both analog techniques and digital algorithms is easy. Even by developing new controller algorithms often a PID controller is used as a benchmark. Subsequently, parameter tuning methods for the PID controller are needed to give good performance and good stability margins for the closed loop system.

Several performance specifications in time domain e.g. as rise time, steady state error and overshoot are related to the open loop system in frequency domain. Our developed PID tuning approach (ALS) calculates the PID parameters with regard to specifications in frequency domain, such as phase margin, cross over frequency and a requested controller phase shape. For this calculation the formulas for systems with real left half plane poles is derived. Finally, the tuning results of our method are demonstrated by applying it to a fill level plant [1]. Results are shown in both frequency and time domain and are compared with a ZN tuned PID controller and a plant pole canceling PID controller applied to the same plant.

2 Frequency domain specifications

In practice PID controllers consists of a Proportional, Integral and Derivative element. The derivative is often filtered by using a first-order low-pass filter to avoid very high gains for high frequency signals. This controller is often referred to as real PID controller or PID controller with filtered derivative or in short as PID-T1. ALS is based on the aspect that a PID-T1, consists of a PI-part and a PD-T1-part in series as shown in equation (1).

$$G_C(s) = \underbrace{K_C \frac{(1+T_{C1}\cdot s)}{s}}_{\textit{PI-part}} \cdot \underbrace{\frac{(1+T_{C2}\cdot s)}{(1+T_N\cdot s)}}_{\textit{PD-T1-part}} \qquad (1)$$

The transfer function of the controller is $G_C(s)$, K_C is the controller gain and T_{C1}, T_{C2}, T_N are controller time constants. This view seizes the idea behind Lead-Lag compensation which includes the advantages of both Lead and Lag compensators.

Figure 1 presents how the bode diagrams of PI and PD-T1 part result in one bode diagram of the real PID controller by forming the geometrical sum of both.

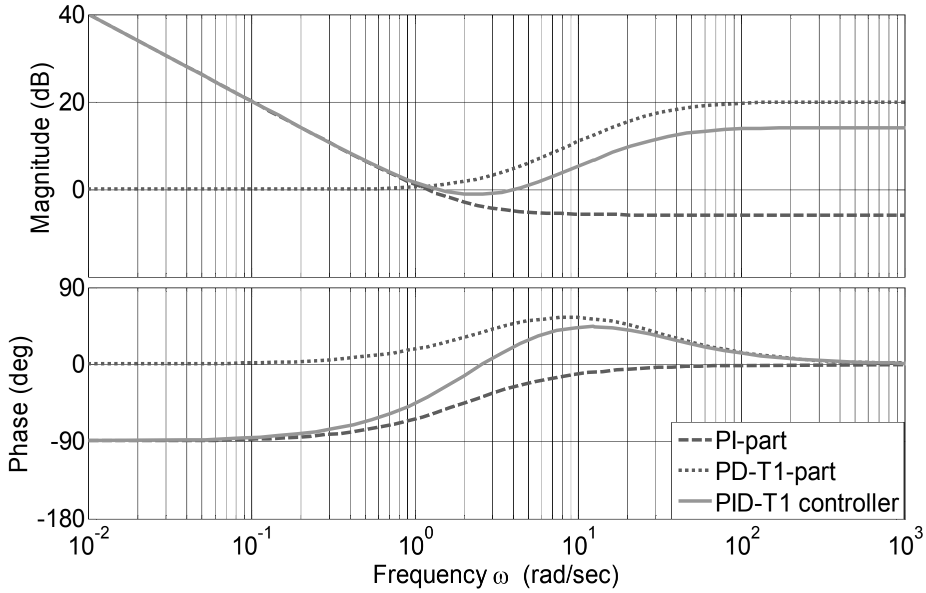

Figure 1: PID-T1 composed of PI-part and PD-T1-part

The most important frequency-domain specification is the phase margin φ_m of the open loop. The phase margin is related to the damping ratio of the closed loop system and thus an important aspect regarding stability. In accordance to [1] a quite normal command response with acceptable overshoot and suitable settling time is expectable for $\varphi_m = 50°$ to 70°. For an aperiodic command response an appropriate phase margin φ_m is 80° to 90°. An acceptable disturbance response is already achievable with $\varphi_m \geq 30°$. A negative open loop phase margin means instability for the closed loop system. The ASL method therefore ensures the open loop phase margin which the user predefines in advance.

The open loop crossover frequency ω_c is referred to as that frequency at which the gain (resp. magnitude) is unity, or 0 dB [2]. It is a criterion for the closed loop speed in time domain.

A good tracking performance of command signals and good attenuation of low-frequency disturbances is achievable with a large open loop gain at low frequencies [4]. As it can be taken from Figure 1 and Equation 1, the low-frequency gain can be increased by the PI-part of the real PID controller. Assuming that the crossover frequency ω_c of an open loop bode diagram is kept constant. Then the PI phase lag $\varphi_{PI}(\omega_c)$, referred to as the phase lag that is contributed too by the PI part at ω_c, affects the value of the controller nominator time-constant T_{C1} and the controller gain K_C as well. Subsequently, the integrating controller action and thus the open loop gain at low frequencies is influenced by $\varphi_{PI}(\omega_c)$. An accurate value for $\varphi_{PI}(\omega_c)$ depends on the process dynamics and the requirement on closed loop performance terms.

In order to avoid the amplification of high-frequency noise, the high-frequency gain of the open loop should be low [4]. In case of actuator saturation such a so-called high-frequency roll-off is also useful to keep the controller output within the operating range of the actuator, even if a large derivative control error is present. The derivative controller action is effectively reduced by the high-frequency roll-off. Criteria for the derivative controller action in frequency-domain are the maximal PD-T1 phase lead $\varphi_{PD-T1_{max}}$, referred to as the maximal phase lead that is contributed too by the PD-T1-part and the frequency $\omega_{\varphi_{PD-T1_{max}}}$, referred to as that frequency at which $\varphi_{PD-T1_{max}}$ occurs in the controller bode plot. Assuming that $\omega_{\varphi_{PD-T1_{max}}}$ is kept constant. Then changing the amount of $\varphi_{PD-T1_{max}}$ is the only way to influence the derivative controller action and the high-frequency roll-off. For processes with distinct dead time it is recommended to set $\varphi_{PD-T1_{max}}$ to zero, because then the derivative action is useless and can even cause unwanted oscillations. In this case a PI controller accrues since the controller denominator time constant T_N compensates the second controller nominator time constant T_{C2}. Furthermore, $\varphi_{PD-T1_{max}}$ should be kept very low if the process consists of numerous dominant poles. This is because a higher order system can be approached by a dead-time and a first order delay element in series. The ratio of the equivalent dead time to the equivalent time constant becomes larger with the number dominant poles. For many processes with numerous dominant poles it makes sense to renounce derivative action and apply a PI controller directly.

ALS is based on the user-defined frequency-domain specifications: phase margin φ_m, PI phase lag $\varphi_{PI}(\omega_c)$ and maximal phase lead $\varphi_{PD-T1_{max}}$. It is applied to calculate the PID parameters in order that the user-defined frequency specifications are fulfilled and the open loop crossover frequency ω_c is maximized. Therefore ALS positions the controller time constants T_{C2} and T_N in a way that $\omega_{\varphi_{PD-T1_{max}}}$ complies ω_c.

The developed PID tuning approach enables the user to shape the open loop bode diagram and thus the time domain performance of the closed loop system in a certain range. Thereby ALS supports by optimizing the closed loop speed with respect to the predetermined frequency domain specifications.

3 ALS Derivation

For maximizing the crossover frequency ω_c the frequency $\omega_{\varphi_{PD-T1_{max}}}$ has to comply with ω_c. Hence,

$$\omega_{\varphi_{PD-T1_{max}}} = \omega_c \tag{2}$$

The ALS method and its equations which are needed to calculate the parameters for a first order time delay process are derived below.

3.1 Determination of ω_c

First the maximum possible closed loop crossover frequency ω_c is determined. Therefore the maximum allowed phase lag of the process at ω_c $\varphi_{plant}(\omega_c)$ is calculated by

$$\varphi_{plant}(\omega_c) = -180° + \varphi_m - \varphi_{PD-T1_{max}} - \varphi_{PI}(\omega_c)\,. \tag{3}$$

With the bode diagram of the plant that frequency is determined at which the plant phase φ_{plant} complies $\varphi_{plant}(\omega_c)$. The determined frequency is the desired open loop crossover frequency ω_c. Now all four parameters of the real PID controller are determined by means of the plant transfer function and the three chosen frequency domain specifications of the closed loop bode diagram.

3.2 Determination of the controller time constants

First the controller time constant T_{C1} is calculated. Therefore the controller PI-part as represented in (1) is rearranged and transformed to frequency domain, such that

$$G_{PI}(j\omega) = \frac{K_C}{j\omega} + K_C \cdot T_{C1}\,. \tag{4}$$

In (4) the PI-part is shown in its additive form (integrating action plus proportional action). Expanding the equation with $\frac{j}{j}$ yields

$$G_{PI}(j\omega) = \underbrace{-\frac{j \cdot K_C}{\omega}}_{Im} + \underbrace{K_C \cdot T_{C1}}_{Re}\,. \tag{5}$$

An equation for T_{C1} as a function of ω and $\varphi_{PI}(\omega)$ is derived if we rearrange Equation (5) as follows:

$$T_{C1} = \frac{-1}{\omega \cdot \tan\{\varphi_{PI}(\omega)\}}\,, \tag{6}$$

It follows from Equation (6) that

$$T_{C1} = \frac{-1}{\omega_c \cdot \tan\{\varphi_{PI}(\omega_c)\}}\,. \tag{7}$$

Equation (7) can be applied since ω_c has been determined in 3.1 and $\varphi_{PI}(\omega_c)$ is predetermined by the user.

Now the controller time constants T_{C2} and T_N are calculated. The following Equations (8) and (9) are transferred from Ogata´s tuning approach for a lead compensator [4]. The constant α is defined as the ratio T_N/T_{C2}.

$$\sin\{\varphi_{PD_{max}}\} = \frac{1-\alpha}{1+\alpha} \tag{8}$$

$$\omega_{\varphi_{PD_{max}}} = \frac{1}{\sqrt{\alpha} \cdot T_{C2}} \tag{9}$$

In order to calculate α Equation (7) is rearranged to

$$\alpha = \frac{1 - \sin\left\{\varphi_{PD-T1_{max}}\right\}}{1 + \sin\left\{\varphi_{PD-T1_{max}}\right\}}. \tag{10}$$

An equation for T_{C2} is derived if we rearrange Equation (9) and substitute $\omega_{\varphi_{PD-T1_{max}}} = \omega_c$.

$$T_{C2} = \frac{1}{\sqrt{\alpha} \cdot \omega_c} \tag{11}$$

With the definition of α as the ratio T_N/T_{C2} we find that

$$T_N = T_{C2} \cdot \alpha \tag{12}$$

3.3 Determination of the controller gain K_C

Finally, an equation to calculate the controller gain K_C needs to be found. Therefore the plant frequency response $F_P(j\omega)$ which is modelled with a series of n first order elements is considered as shown in equation (13).

$$F_p(j\omega) = \frac{K_p}{\prod_{i=1}^{n}\left(1 + T_i \cdot j\omega\right)} \tag{13}$$

The open loop frequency response follows from the series of plant and controller frequency response as it is outlined in Equation (14).

$$F_C(j\omega) = \frac{K_C \cdot K_p\left(1 + T_{C1} \cdot j\omega\right)\left(1 + T_{C2} \cdot j\omega\right)}{j\omega \cdot \left(1 + T_N \cdot j\omega\right) \cdot \prod_{i=1}^{n}\left(1 + T_i \cdot j\omega\right)} \tag{14}$$

The magnitude $\left|F_0(j\omega)\right|$ can be determined by

$$\left|F_C(j\omega)\right| = \frac{\sqrt{K_C{}^2 K_p{}^2 + K_C{}^2 K_p{}^2 T_{C1}{}^2 \omega^2}\sqrt{1 + T_{C2}{}^2\omega^2}}{\sqrt{\omega^2 + T_N{}^2\omega^4} \cdot \prod_{i=1}^{n}\sqrt{1 + T_i{}^2\omega^2}}. \tag{15}$$

In order to achieve an expression for the controller gain K_C Equation (15) is rearranged such that

$$K_C = \sqrt{\frac{\left|F_C(j\omega)\right|^2 \cdot \left(\omega^2 + T_N{}^2\omega^4\right) \cdot \prod_{i=1}^{n}\left(1 + T_i{}^2\omega^2\right)}{\left(K_p{}^2 + K_p{}^2 T_{C1}{}^2\omega^2\right)\left(1 + T_{C2}{}^2\omega^2\right)}}. \tag{16}$$

Finally, with $\left|F_C(j\omega_c)\right| = 1$ at ω_c Equation (16) can be applied to

$$K_C = \sqrt{\frac{\left(\omega_c^{\,2} + T_N^{\,2}\omega_c^{\,4}\right)\cdot \prod_{i=1}^{n}\left(1+T_i^2\omega_c^{\,2}\right)}{\left(K_p^{\,2} + K_p^{\,2}T_{C1}^{\,2}\omega_c^{\,2}\right)\left(1+T_{C2}^{\,2}\omega_c^{\,2}\right)}}\,. \tag{17}$$

In summary the ALS tuning method consists of six steps:

1. Calculation of $\varphi_{plant}(\omega_c)$ with Equation (2)
2. Determination of ω_c by using the bode diagram of the plant as explained in 3.1
3. Calculation of T_{C1} with Equation (6)
4. Calculation of the factor α with Equation (9)
5. Calculation of T_{C2} and T_N with the Equations (10) and (11)
6. Calculation of K_C with Equation (16)

4 Comparison with other tuning methods

In order to show the results that can be achieved with ALS the parameter tuning is shown by a concrete example and compared with a ZN tuned PID controller and a PID controller based on plant pole cancellation [1]. The example process is a fill level plant which has the transfer function (18) [1].

$$G_P(s) = \frac{4.57}{(1+175.4\cdot s)(1+s)(1+0.47\cdot s)^2} \tag{18}$$

The high-frequency noise is already damped by the low-pass character of the actuator [1]. For this reason the maximum PD-T1 phase lead of the ALS tuned controller is chosen appropriately. The ZN tuned PID controller as well as the pole cancelling PID controller in subchapter 4.2 and 4.3 are even tuned as ideal PID controllers without additional low-pass filter.

In order to get a suitable close loop command response and a certain robustness against model uncertainties the open loop bode diagram should have the phase margin $\varphi_m \approx 60°$. Furthermore, the fill level has to be controlled as fast as possible for a specified tolerance $\Delta x_s = \pm 5\%$ around the reference.

4.1 ALS tuned PID controller

The following frequency-domain specifications were predetermined with which the given tolerance Δx_s is utilized to achieve a very fast close loop performance.

- Phase margin $\varphi_m = 60°$
- PI phase lag $\varphi_{PI}(\omega_c) = -30°$
- Maximum PD-T1 phase lead $\varphi_{PD-T1_{max}} = 85°$

The resultant PID-T1 controller which is tuned by ALS is given by

$$G_C(s) = 0.976\frac{(1+26.38\cdot s)(1+2\cdot s)}{s(1+0.05\cdot s)}. \tag{19}$$

The results in both frequency and time domain are shown below in Figure 2 and Figure 3.

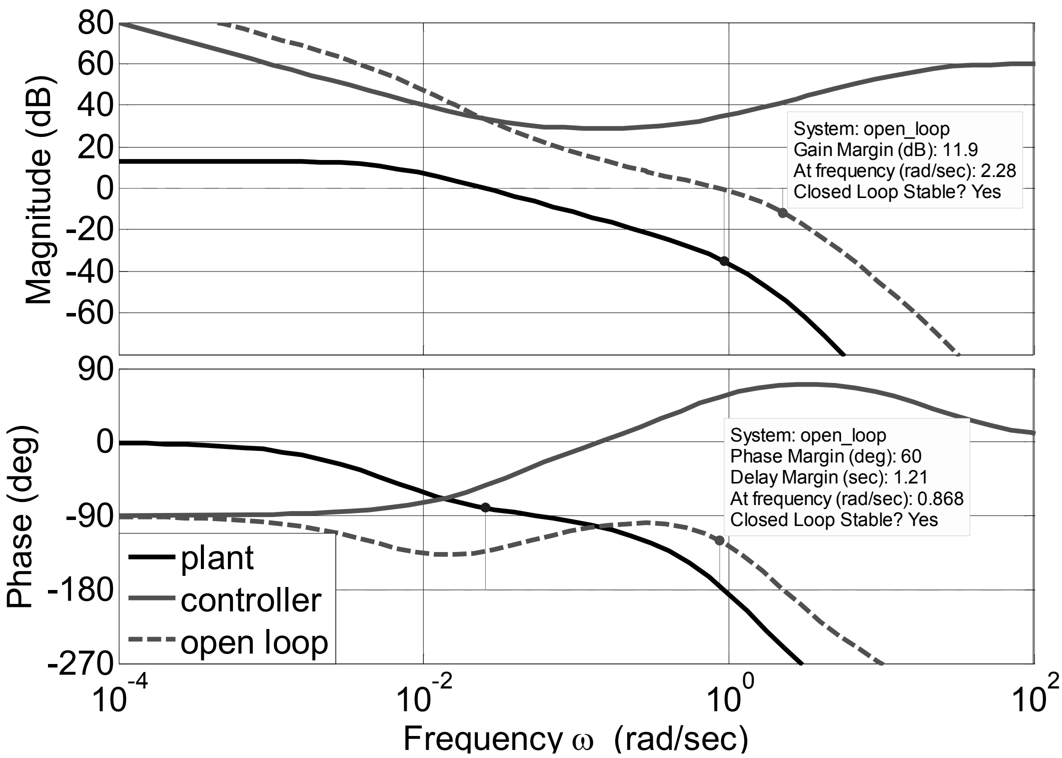

Figure 2: Bode diagrams of the plant simulation, the ALS tuned PID-T1 controller and the open control loop

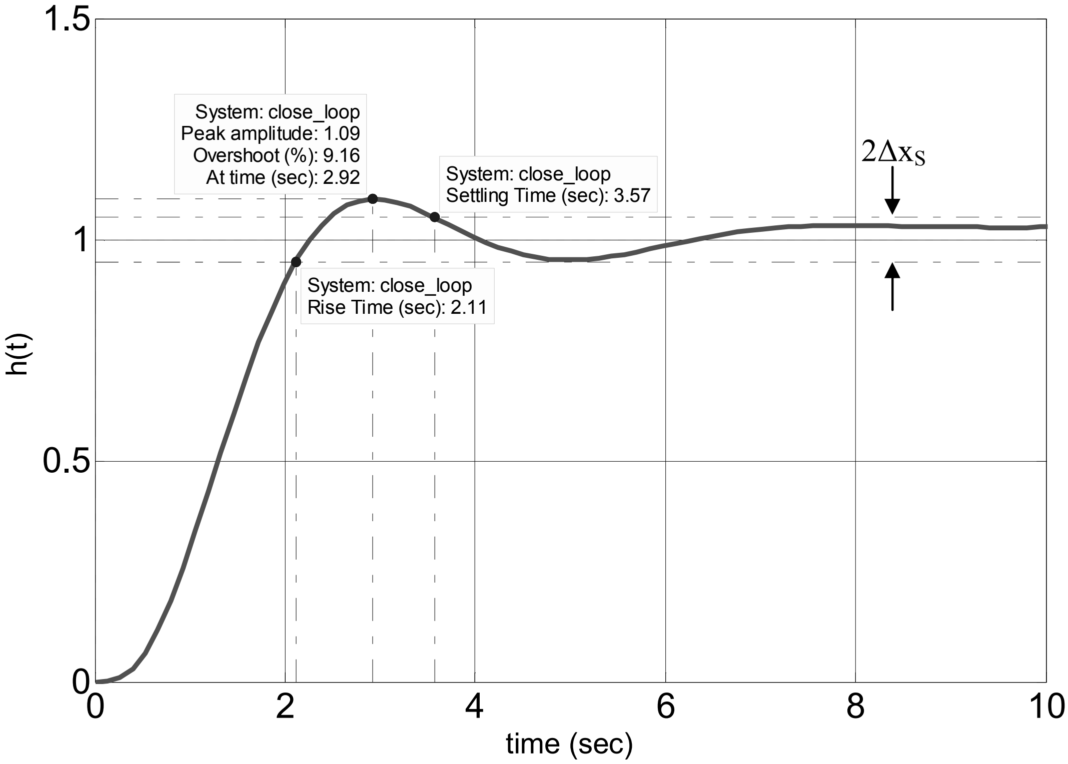

Figure 3: Unity closed loop reference step response simulated with the ALS tuned PID controller

4.2 ZN tuned PID controller (open loop method)

The resultant PID-T1 controller which is tuned by the ZN open loop method is given by

$$G_C(s) = 26.65\left(1 + \frac{1}{3.71 \cdot s} + 0.93 \cdot s\right). \tag{20}$$

The ZN open loop method is not based on bode diagrams but only on an empirical determination of the controller parameters from the process step response. The ZN tuning results are shown in time domain (Figure 4).

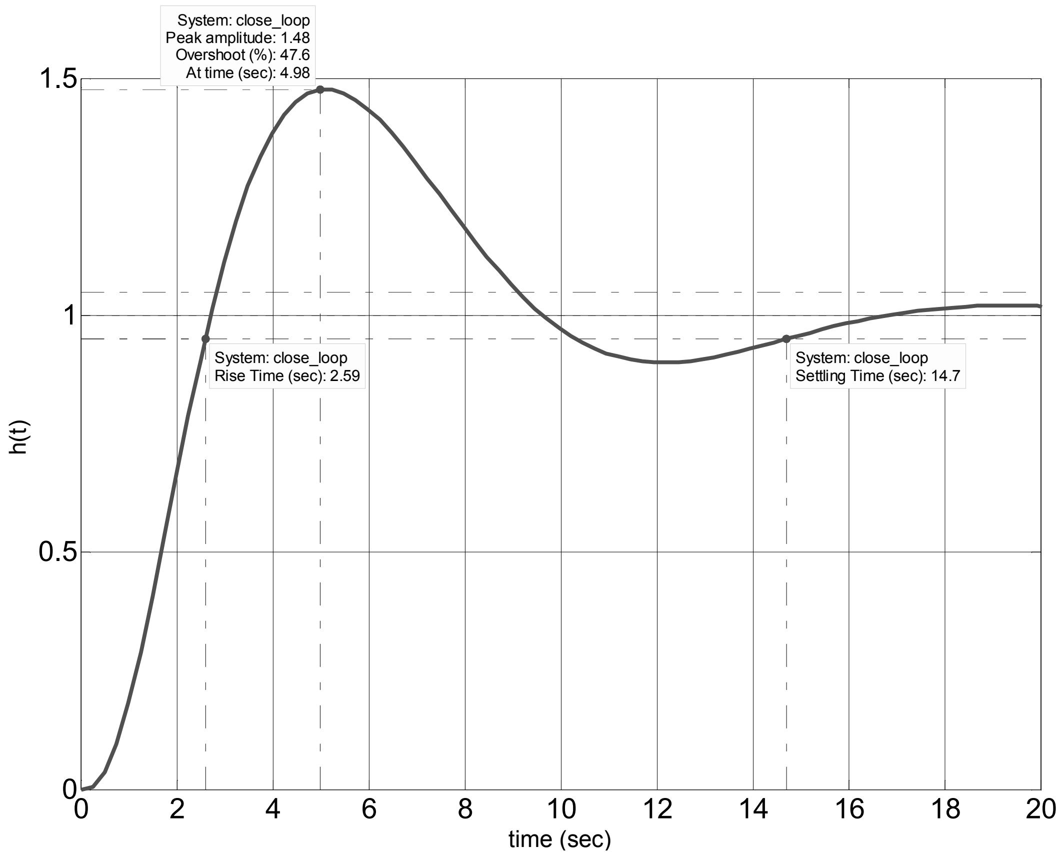

Figure 4: Unity closed loop reference step response simulated with the PID controller tuned by ZN open loop method

4.3 PID controller tuned by pole cancellation

The resultant PID-T1 controller which is tuned by pole cancellation is given by

$$G_C(s) = 0.134\frac{(1+175.4\cdot s)(1+s)}{s}. \tag{21}$$

The results in both frequency and time domain are shown below in Figure 5 and Figure 6.

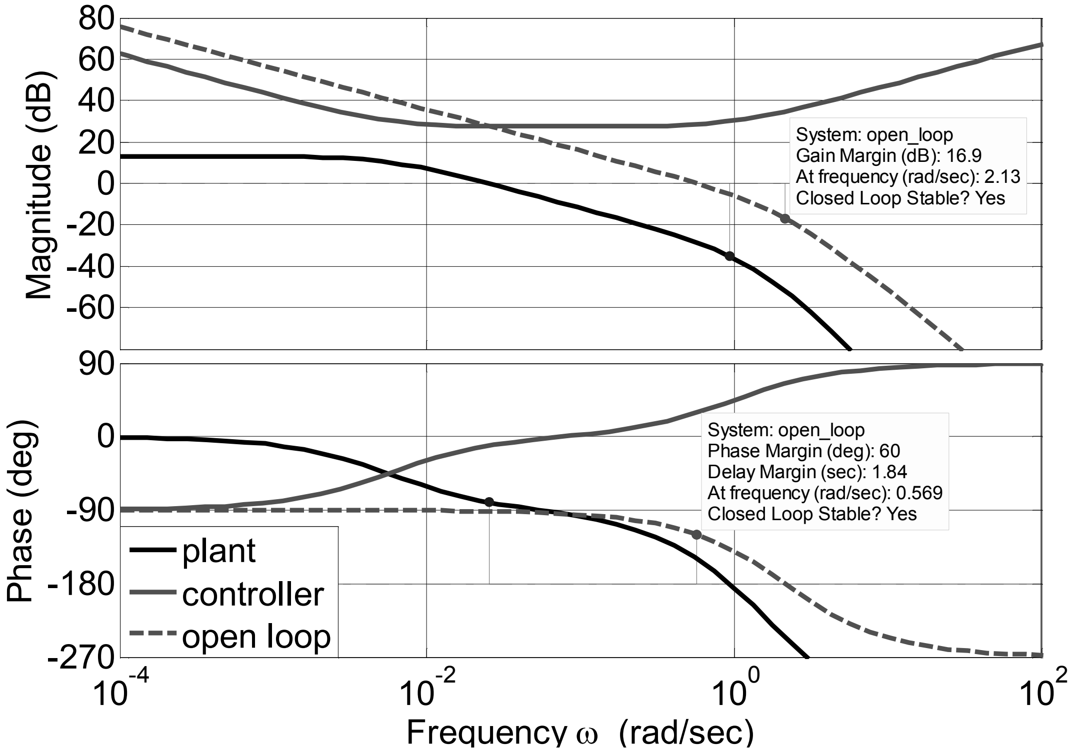

Figure 5: Bode diagrams of the plant simulation, the PID controller tuned by plant pole cancellation and the resultant open control loop

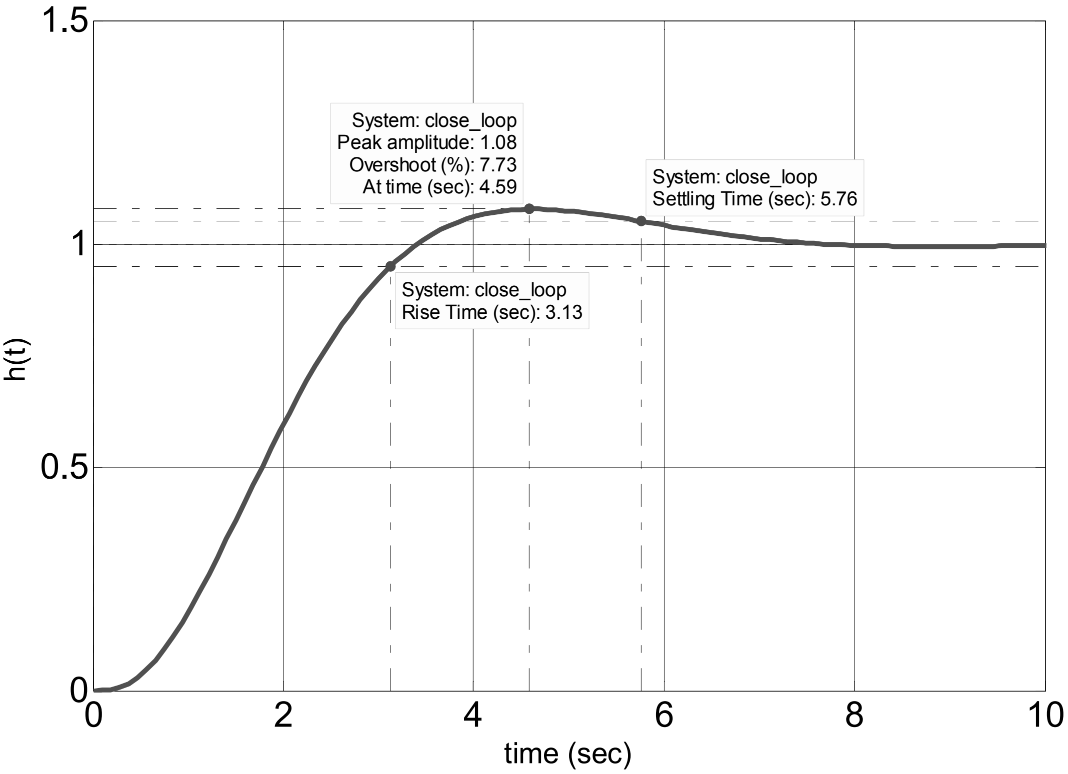

Figure 6: Unity closed loop reference step response simulated with the PID controller tuned by plant pole cancellation

5 Conclusion

The advantage of our ALS method compared to empirical tuning methods like e.g. ZN is that a desired phase margin value can be assured. The results in subchapter 4.2 demonstrate that the ZN tuned PID controller obtains a phase margin less then the here requested 60°. Subsequently, the settling time of the closed loop system is large due to vast oscillations and the closed loop system lack robustness. The advantage of ZN methods is that only little process information is required whereas the ALS method needs the exact model of the process.

The PID controller which is tuned by pole cancellation offers the required phase margin too and a good performance is obtained with short rise time, acceptable overshoot and suitable settling time. With the ALS method tuned PID controller the response speed of the closed loop is better, which is characterized by the fast rise time and short settling time. This is because the given tolerance about $\pm 5\%$ are utilized. This is only possible due to the appropriately chosen and balanced frequency-domain specifications PI phase lag and maximum PD-T1 phase lead. The advantage of a possible fine tuning of the ALS method is disadvantage of it as well, as sometimes several trials are needed until all closed loop performance specifications are sufficient fulfilled.

In further studies we will test the ASL method in real applications with different plant models. Furthermore we will compare the method to cases, when cancellations methods are disastrous because the canceled factors are unstable [4]. A similar PD controller tuning method for systems with integral behavior is planned. Furthermore, we are going to establish a graphical user interface for a PID tuning by ALS, which will be running on a web server on our homepage.

Literature

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