# Optimum Spacing of Solar Modules for Two Axis Tracking

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## 1. INTRODUCTION

In Middle-Europe a significant increase in solar energy production can be achieved by applying solar tracking in photovoltaic (PV) farms. According to the technical literature between 20% (Zsiborás) and 40% (www.astrasun.hu) plus energy has been measured with solar tracking compared to energy production of fixed PV farms e.g. in Hungary. However this high achievement is only true if a single module arrangement is rotated as a unit with no neighbouring modules.

How much plus energy can be expected in case of several rotated modules placed near to each-other compared to fixed panels occupying the same surface? There are papers in the technical literature partly answering this question, e.g. how to optimize solar field design for single axis tracking (Appelbaum) and for stationary collectors (Weinstock).

If the basis for the comparison is the total surface occupied by a solar field made up of PV modules placed densely near to each-other, then the plus energy can be less than the expected maximum, since the modules begin to shadow their neighbouring panels when being rotated. This shadowing can be reduced by applying gaps between the rotated units, however than a part of the total surface is inactive for energy production thus lowering the resulted energy yield.

This paper proposes a mathematical method to determine the optimum gap dimensions between the rotated modules both in East-West and North-South direction which arrangement assures maximum solar energy produced by two-axis tracking on a certain area.

## 2. DIMENSIONS AND PRECONDITIONS

Dimension of the PV module rotated as one unit is $a$ in East-West direction with a gap $c$ between the neighbouring modules and $b$ in North-South direction with a gap $d$ between them (Fig. 1). Thus the total area occupied by one module is $A = (a+c)(b+d)$ and this area serves as basis for the comparison of energy yields in case of different $c$ and $d$ values. The active part of the basis area is $A_a = ab$ producing PV energy.

For the first step of developing the mathematical method the following preconditions are taken into account: 1. Modules are placed on a horizontal surface. 2. An internal module will be analysed surrounded by other modules in every directions, i.e. to the north, north-east, east, south-east, south, south-west, west and north-west. 3. Modules face always perpendicular to the direct solar radiation, so they rotate and tilt continuously. 4. Only clear weather is taken into account without any clouds. 5. Shadows have always sharp edges on the panels in any distances only core shadow is taken into account. 6. The method is valid only locations with latitudes between the north polar circle and the Tropic of Cancer.

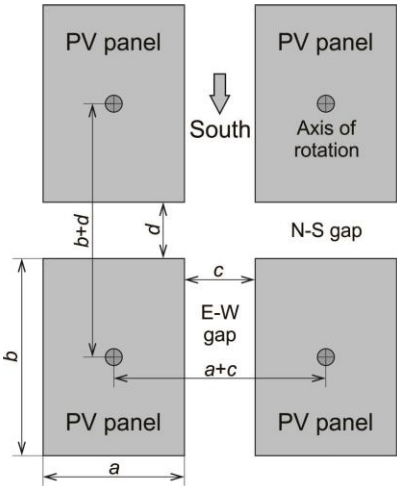


Fig. 1. Dimensions of the PV modules laying horizontally.

## 3. EQUATIONS OF THE CALCULATION

### *3.1 Equations of the Sun's path*

Since the Sun's path is symmetrical to the solar south direction it is enough to analyse one half of its path. Thus the panels are rotated from their position facing south to their direction at sunrise increasing the so called $\beta$ hour angle. The dependence of $\alpha$ solar elevation angle on the $\beta$ hour angle is

$$\sin\alpha = \sin\Phi\sin\delta + \cos\Phi\cos\delta\cos\beta \quad (1)$$

where $\Phi$ is the latitude of the location of the PV panels (48.21° in case of Vienna) and $\delta$ is the declination at the given day which can be calculated with

$$\delta = -23.44\cos\left(\frac{360}{365}\,(N+10)\right) \quad (2)$$

where $N$ is the number of the given day. $N = 172$ in case of the 21[st] of June. On this day the declination is maximum, i.e. $\delta_{06.21} = 23.44°$. Then with the help of (1) the hour angle of the sunrise is $\beta_{Sr} = 119.02°$ for the latitude of Vienna.

### *3.2 Equations for calculating the shadowed area*

The shadow thrown by a neighbouring panel onto the PV panel under investigation has a horizontal dimension $e$ and a vertical dimension $f$. Geometry of the panels is shown in Fig. 2 for hour angles $90° \le \beta \le \beta_{Sr}$ for the calculation of dimension $e$ and in Fig. 3 for the calculation of $f$.

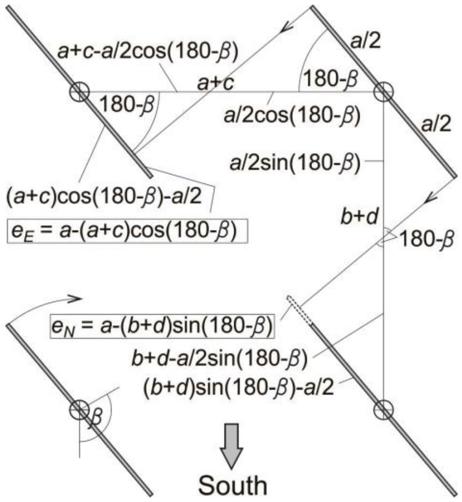


Fig. 2. Upper view of the panels for $90° \le \beta \le \beta_{Sr}$.

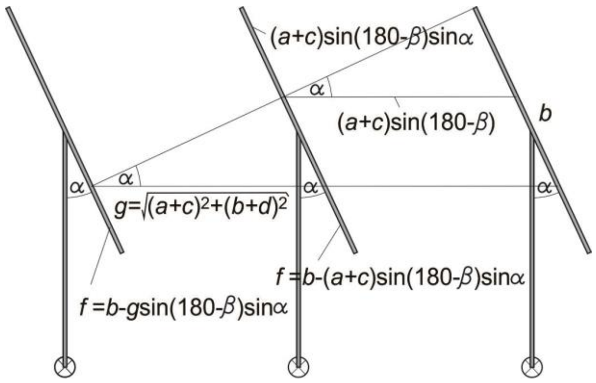


Fig. 3. Lateral view for calculating $f$ for $90° \le \beta \le \beta_{Sr}$

Within the frame of this extended abstract only the shadow thrown by the Eastern neighbour and the gap dimension $c$ is analysed. Horizontal dimension of the shadow thrown by the Eastern panel for hour angles $90° \le \beta \le \beta_{Sr}$ is

$$e_{E2} = a - (a+c)\cos(180-\beta) \quad (3)$$

and for hour angles $0° \le \beta < 90°$

$$e_{E1} = a - (a+c)\cos(\beta). \quad (4)$$

Vertical dimension of the shadow thrown by the Eastern panel in the range of $90° \le \beta \le \beta_{Sr}$ is

$$f_{E2} = b - (a+c)\sin(180-\beta)\sin\alpha \quad (5)$$

and the vertical dimension of the shadow thrown by the Eastern panel in the range of $0° \le \beta < 90°$ is

$$f_{E1} = b - (a+c)\sin\beta\sin\alpha. \quad (6)$$

## 4. CALCULATION OF THE OPTIMUM GAP SIZE

For this first step of analysis dimensions $a$ and $b$ of the panel result unit surface, i.e. $A_a = ab = 1$ not taking account dimensions. The part of the surface of the panel exposed to direct sunlight produces unit power and the shadowed surface part only $0.1ef$ power (https://wikipedia.org.wiki.Solar-tracker). Then this value is divided by the total area $A$ occupied by the panel. Thus the resulted normalized power produced by the panel at a certain angle $\beta$ is

$$P(\beta) = \frac{ab - 0.1\, e(\beta) f(\beta)}{(a+c)(b+d)}, \quad (7)$$

where $d = 0$ for this analysis. Then the power values are summarized for every integer $\beta$ angle values being proportional to time. This result is signed with letter $W$, referring to the PV energy production.

Calculations has been performed with the mathematical software MAPLE. In Fig. 4 dependence of $W$ on the gap dimension $c$ is shown. The found maximum value belongs to $c = 0.45$ (approximately), i.e. 0.45 times $a$.

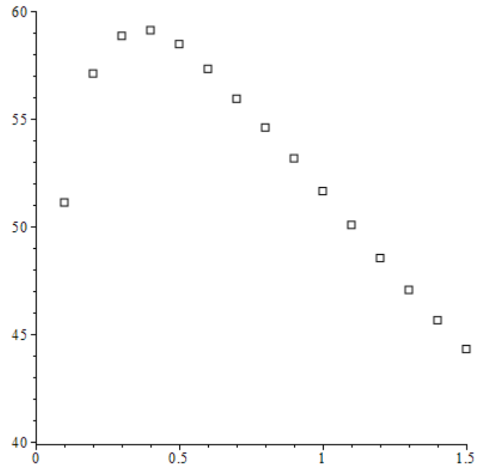


Fig. 4. Plot of $W$ vs gap dimension $c$.

## 5. CONCLUSIONS AND FURTHER WORK

Results of the calculation verify the expectation that there is an optimum gap size between PV panels even in case of taking into account only two panels. As next steps the other neighbouring panels and the dependence of the irradiation intensity on the hour angle will be taken into account.

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