Modelling of the Self-Propelled Vibro-Impact Capsule in Small Intestine ⋆

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1. INTRODUCTION

The technology of capsule endoscopy, which employs a swallowable device propelled by gastrointestinal peristalsis through small intestinal tract to transmits video images, has been used in the past decade for evaluating gastrointestinal bleeding, inflammation, tumours, and some other diseases, see McCaffrey et al. (2008); Koulaouzidis et al. (2013) However, the bottleneck issue of such technology is the lack of motion control, i.e. the capsule cannot settle down at a suspected area for longer period or take a reverse travel within the tract, causing missing images of symptom, which has consumed massive doctor's time and oversight of diseases.

This work concerns this issue by employing the so-called vibro-impact capsule technique (see e.g. Chernousko (2002); Liu et al. (2013); Jiang and Xu (2017)), which utilizes internal vibration and impact forces for bidirectional rectilinear self-propelled driving. Mathematical modelling of the vibro-impact capsule system will be considered in the environment of small intestinal tract, and a complete analysis of the capsule's performance in terms of its progression speed and energy efficiency will be carried out.

2. MATHEMATICAL MODELLING

In this work, we consider the two-degrees-of-freedom dynamical capsule system as shown in Fig. 1(a), where a movable internal mass m_1 is driven by a harmonic force with magnitude P_d and frequency Ω. The internal mass interacts with a rigid capsule m_2 via a linear spring with stiffness k and a viscous damper with damping coefficient c. The capsule has a cylindrical body with a hemispherical head and tail. Impact between the internal mass and a weightless plate connected to the capsule through a secondary spring with stiffness k_1 may occur, once their relative displacement $x_1 - x_2$ is larger or equal to the gap g_1, where x_1 and x_2 are the absolute displacements of the internal mass and the capsule, respectively.

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2.1 Resistance

As the diameter of the capsule is larger than the inner diameter of the small intestine, the capsule stretches the intestinal tract to yield hoop stress. This hoop stress causes normal and frictional forces on the capsule yielding environmental resistance which prevents the motion of the capsule. In addition, the gravity of the capsule which exerts normal pressure on the intestinal tract also adds additional value to the resistance. It is therefore that the overall resistance on the capsule can be written as

$$F_r = F_{\text{hoop}} + F_{\text{gravity}}, \tag{1}$$

where F_{hoop} and F_{gravity} represent the resistances introduced by hoop stress and capsule gravity, respectively. As depicted in Fig. 1(b), the resistance due to the hoop stress can be given as

$$F_{\text{hoop}} = -\,\text{sign}(v_2)(F_{Hp} + F_{Tp} + F_{Hf} + F_{Bf} + F_{Tf}), \tag{2}$$

where v_2 is the capsule speed, F_{Hp} and F_{Tp} are the normal pressures of the intestine on the capsule head and tail, and F_{Hf}, F_{Bf} and F_{Tf} are the frictional forces exerted on the head, the body, and the tail of the capsule, along the axial direction of the capsule, respectively. As the cross section of the small intestine is expanded by the capsule yielding tensile stress, the hoop stress depends on the geometric deformation of the intestinal wall. The geometric parameters of the capsule are shown in Figure. 1(b), where L is the length of the capsule, R_c is the radius of the head, the body, and the tail, R_i is the original inner radius of the intestinal tract, ϕ_c is the angle of the point from where the intestine tract starts to surround the capsule, and x_c is the distance from the contact point to the centre of the head (or the tail).

2.2 Equations of motion

As depicted in Fig. 1, a periodic external excitation, $P_d \cos(\Omega t)$, is applied on the inner mass m_1 to drive the capsule m_2. The inner mass interacts with the capsule via a damped spring at the tail and a secondary spring at the head of the capsule. Due to the gap between the mass

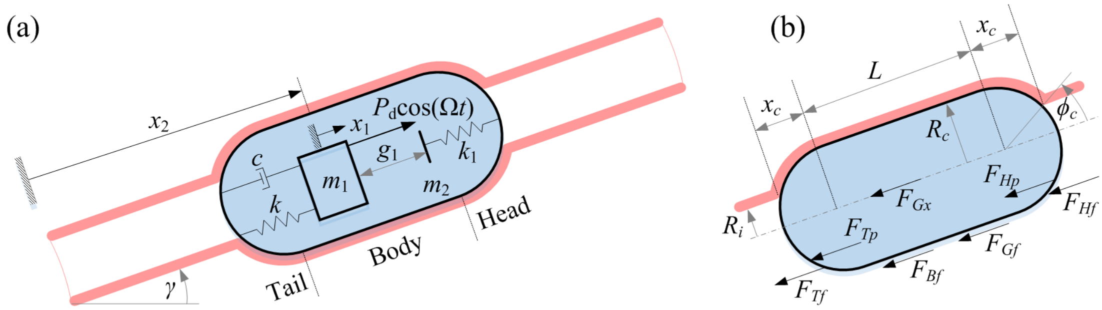

Fig. 1. (a) Physical model of the vibro-impact capsule in small intestine. (b) Resistance forces and geometric parameters of the capsule. The capsule is depicted in cyan, and the intestinal tract is presented in light red.

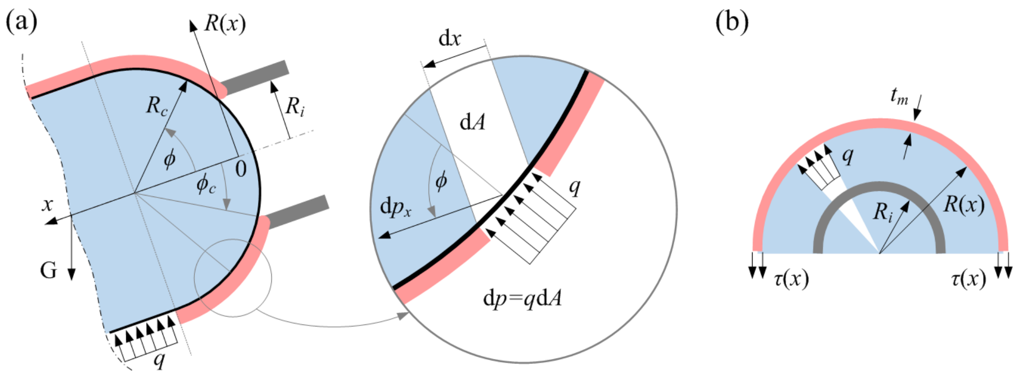

Fig. 2. (a) Hoop stress on the head and the body of the capsule. (b) Cross section of the intestinal tract. The intestinal tract without stretch is depicted in grey, and the one with stretch is shown in light red.

and the secondary spring, g_1, the interaction between m_1 and m_2 keeps switching between two phases: no contact $(x_1 - g_1 - x_2 < 0)$ and contact $(x_2 - g_1 - x_1 \geq 0)$. Therefore, the mutual interactive force between the inner mass and the capsule can be written as

$$F_i = -c(\dot{x}_1 - \dot{x}_2) - k(x_1 - x_2) - H_1 k_1 (x_1 - g_1 - x_2), \quad (3)$$

where H_1 is the Heaviside function given by

$$H_1 = H(x_1 - g_1 - x_2). \quad (4)$$

Here, a detailed consideration of these switching phases can be found in Liu et al. (2013). Finally, the comprehensive equations of motion for the vibro-impact capsule system are written as

$$\begin{aligned} \dot{x}_1 &= v_1, \\ \dot{v}_1 &= \frac{1}{m_1}[P_d \cos(\Omega t) + F_i] - g \sin \gamma, \\ \dot{x}_2 &= v_2, \\ \dot{v}_2 &= -\frac{1}{m_2}[F_i - F_{\text{hoop}} - F_{\text{gravity}}] - g \sin \gamma. \end{aligned} \quad (5)$$

3. CONCLUDING REMARKS

Modelling of the self-propelled vibro-impact capsule system moving in a small intestinal tract was studied in this paper. Our studies focused on exploring the dynamics of the system and its performance in terms of average velocity and energy efficiency under variations of different system and control parameters, such as the forcing frequency and magnitude, the natural frequency of the inner mass, the contact gap between the inner mass and the secondary spring, and the capsule's radius and length.

Future works include prototype design and fabrication, test rig design, and experimental testing of the capsule prototype. Numerical studies in this paper will support the design and fabrication of the capsule prototype, and an artificial intestinal environment will be created for model validation. Research findings along this direction will be reported in a separate publication in due course.

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