Modeling of Heat Transfer in Controlled Processes for Cylindrical Bodies

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1. INTRODUCTION

Reliable modeling is needed when different control strategies are applied to distributed parameter systems, see Butkovsky (1969); Chernousko et al. (1996). Problems of heat transfer control for different simple bodies like cylinders, rods, and bars were recently considered in a large number of papers, e.g., in Kersten et al. (2014); Rauh et al. (2015a,b). In Kostin et al. (2017), an optimal control of the heating of a metal bar was presented. The current paper considers modeling the active heating of a cylindrical body. The cylinder may be aligned either vertically or horizontally. Two Peltier elements provide heat fluxes on both ends of this body. A corresponding experimental setup is available at the Chair of Mechatronics, University of Rostock, Germany. Experimental results obtained from this test rig are used for the identification of the system parameters. Simulations with the identified parameters match the measurements with high accuracy.

2. DESCRIPTION OF THE MODEL

Let us consider the model of a cylinder insulated thermally on both end faces. The cylinder is given by the domain $\Omega = [0, l] \times [0, r_0] \times [0, 2\pi]$, where l is its length and r_0 is its radius. It has free contact with the ambient air over its lateral surface. The cylinder is heated up or cooled down from both end faces by two Peltier elements, which produce two independent heat fluxes F_1 and F_2. The corresponding system of equations is given by

$$\begin{cases} \rho c_{\mathrm{p}} \dfrac{\partial \vartheta}{\partial \tau} = \lambda \left(\dfrac{\partial^2 \vartheta}{\partial z^2} + \dfrac{1}{r} \dfrac{\partial}{\partial r} \left(r \dfrac{\partial \vartheta}{\partial r} \right) + \dfrac{1}{r^2} \dfrac{\partial^2 \vartheta}{\partial \varphi^2} \right) \quad \text{in } \Omega, \\ \dfrac{\partial \vartheta}{\partial n}\Big|_{r=r_0} = \alpha(z, \varphi)(\vartheta(z, r_0, \varphi, \tau) - \vartheta_{\mathrm{a}}(\tau)), \\ \dfrac{\partial \vartheta}{\partial n}\Big|_{z=l} = F_1(u_1(\tau)), \\ \dfrac{\partial \vartheta}{\partial n}\Big|_{z=0} = F_2(u_2(\tau)), \\ \vartheta|_{\tau=0} = \vartheta_{\mathrm{b}}^0. \end{cases} \tag{1}$$

The voltages applied to the elements, $u_1(\tau)$ and $u_2(\tau)$, are adjusted during the active heating, where the functions $u_1(\tau)$, $u_2(\tau)$ are defined for the time interval $\tau \in [0, T]$. The temperature $\vartheta(z, r, \phi, \tau)$ of the cylinder is equal to ϑ_{b}^0 at the time instant $\tau = 0$. Temperature measurements are available at several distinct locations on the cylinder's circumferential surface. The model has been derived on the basis of the partial differential equation for heat transfer.

When specifying boundary conditions on the circumferential surface of the cylinder, we address the heat exchange with the ambiant air and model the Peltier elements. The latter were studied in Rauh et al. (2015a). The results from Rauh et al. (2015a) are employed to construct the Peltier element operation model, i.e., to define flux functions $F_1(\cdot)$, $F_2(\cdot)$. A series of experiments was performed for this purpose.

The process of heat transfer for a cylindrical body was investigated in Knyazkov et al. (2017). The corresponding analytical results are used in the current paper for estimating the heat exchange with the ambient air. The heat transfer coefficient in the model is estimated both theoretically and experimentally, where the analytical and experimental estimates are in a good agreement (see Fig. 1).

3. RESULTS AND OUTLOOK

The active heating for the aluminum cylinder is modeled by the finite element method and then compared to measurements. Experimental data are only available for the vertically aligned aluminum cylinder. All other results for the different alignments and materials presented in the current paper are outcomes of pure simulations.

An example of a simulation performed for a glass cylinder is described below. The influence of the variability of the heat transfer coefficient along the body surface is investigated. The results of modeling of the controlled heat transfer for the horizontally aligned glass cylinder are shown in Fig. 2. Solid lines show the temperatures at the points $L_i = (\frac{l}{2}, r_0, \frac{\pi}{4}(i-1))$, $i = 1, \ldots, 5$, for the constant heat transfer coefficient $\overline{\alpha}$, while the dashed lines stand for

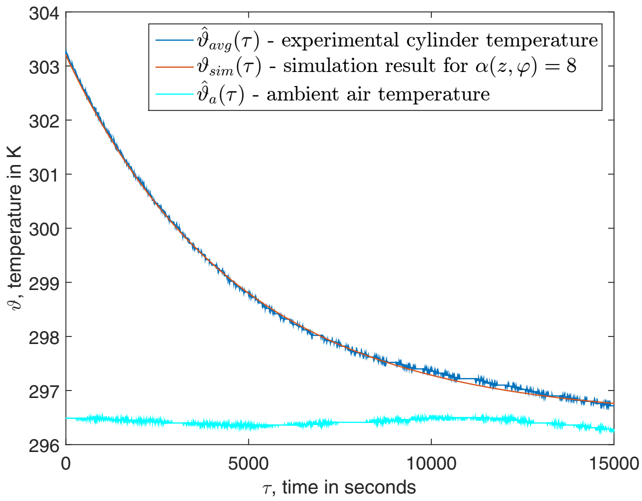

Fig. 1. Simulated average temperature $\vartheta_{\mathrm{sim}}(\tau_k)$ fits the experimental averaged temperature $\hat{\vartheta}_{\mathrm{avg}}(\tau)$ for the constant heat transfer coefficient $\alpha(z,\varphi) = 8$. Here, $\hat{\vartheta}_{\mathrm{avg}}(\tau)$ is the average of experimentally measured temperatures for the aluminum cylinder. The experimental ambient air temperature $\hat{\vartheta}_{\mathrm{a}}(\tau)$ is shown by the cyan curve

the specified dependency $\alpha_{\mathrm{hor}}(\varphi)$ of the convective heat transfer coefficient on the azimuth angle. The difference in temperature is significant when the dependency of the convective heat transfer coefficient on the azimuth angle is taken into account as compared with the case when a constant averaged value of this coefficient is utilized.

The results show that the obtained temperature can differ by several K when the body is heated by 10–15 K. This demonstrates that the variability of the heat transfer coefficient along the surface of the body should be taken into account when considering a problem of heat transfer in materials that have a small conductivity.

One of the future goals on the topic is to consider the case of heating with Peltier elements that have passively or actively cooled units from the side that is not in direct contact with the cylinder. An experiment with a glass cylinder should be performed that can justify the influence of the heat transfer coefficient's variability on the temperature distribution inside the body. It will be very useful in this case to register not only the temperature in a fixed number of surface points, but also the distribution of the surface temperature during the heating process. This can be done by optical measurement techniques.

The designed model and the developed computational approach can be used when modeling the heat transfer for more sophisticated control laws. An example of optimal control for the described system can be found in Gavrikov and Kostin (2017). However that model did not take into account the position dependency of the heat transfer coefficient. The simplified analytical expressions for the heat transfer coefficient from Knyazkov et al. (2017) can be applied for an analytical Fourier analysis or for control design when the dependency of the coefficient on the position on the lateral surface of the cylinder is taken into account.

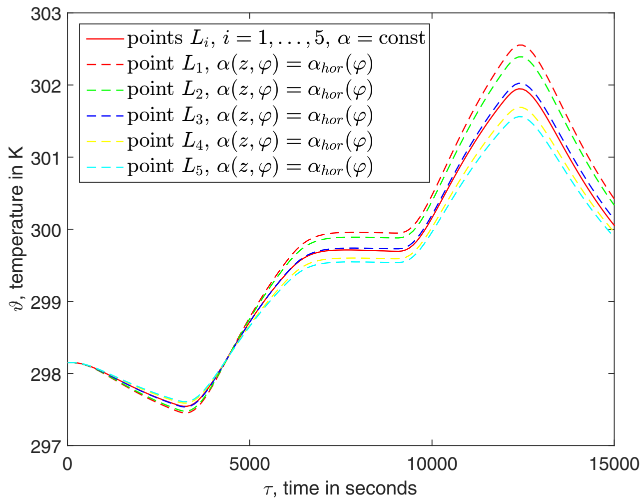

Fig. 2. Heating simulation for the horizontally aligned glass cylinder for two cases: $\alpha(z,\varphi) = \overline{\alpha}$ and $\alpha(z,\varphi) = \alpha_{\mathrm{hor}}(\varphi)$

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