ACTIVE CONTROL FOR THE RE-ENTRY OPERATION OF FLEXIBLE RISERS

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Abstract.

This paper presents an active control dedicated to a re-entry problem found in the offshore oil industry. The re-entry operation consists in connecting the bottom of a very long pipeline to the wellhead, by dynamically modifying the pipeline top position, which is linked to a floating device (vessel or platform). These long pipelines are usually called risers, because they are used to rise the drilling mud or the hydrocarbons from the wellhead to the platform. Nowadays the re-entry operation is done manually. The use of an active control intends to reduce the operation time, and to make it possible even under bad weather conditions. The considered offshore structure can be analyzed as a cable submerged in a flow. A convenient model is given by the Bernoulli's historical cable equation, completed with a damping factor, that linearly depends on the structure speed. The damping factor is developed in series around zero, to get an approximate solution. The corresponding model turns out to be differentially flat[1], a property directly used in the control design, providing an extension to previous works of Petit and Rouchon [2], Thull et al [3], and Sabri [4]. This paper presents an overview of the results of [5]. Furthermore it contains material concerning a new tracking system, that uses the system inversion to define the feedback control.

Introduction

The re-entry operation consists in positioning the riser bottom above the wellhead in order to connect them. To reach this goal, it is necessary to move the riser bottom to the wellhead as fast as possible and to make it stop accurately above the wellhead. The main idea is to define an open loop trajectory to move the riser bottom from its initial position to the wellhead. The closed loop is computed by tracking this prescribed trajectory for the riser bottom.

Nowadays, the re-entries are done by manual control. The interests of using an automatic control are to reduce the operation time and to extend the range of meteorological conditions under which the connection is possible. Actually, because of bad weather, a drilling ship or a FPSO (floating, production, storage and offloading unit) can wait several days until the manual reentry operation becomes possible. The final economic benefit of an active control is linked to the possibility to reduce the global time necessary to drill an offshore well or to restart the production of a production well.

1 Governing equations

Generally, offshore structures are slender and their transverse displacements are small when compared to their height, so they can be analyzed as a linearized Euler-Bernoulli beam with a constant section, under an axial traction plus external forces from the fluid [6]:

$$m_s \frac{\partial^2 \Upsilon}{\partial t^2} = -EJ \frac{\partial^4 \Upsilon}{\partial z^4} + \frac{\partial}{\partial z}\left(T \frac{\partial \Upsilon}{\partial z}\right) + F(z,t) \tag{1}$$

Equation (1) represents the dynamic behavior of the structure in the direction of its main horizontal displacement. The variable Υ represents the structure displacement in this direction and is a function of two parameters: the time t and the height from the seabed z. The other variables are: the structure linear mass m_s, the mechanical tension T, the Young's modulus E, the second moment of inertia J, and the hydrodynamic forces in this direction, represented by F. The Morison's equation defines the hydrodynamic forces associated to a relative displacement of a submerged body in a fluid (see [6]):

$$F(z,t) = -m_F \frac{\partial^2 \Upsilon}{\partial t^2} - \mu \frac{\partial \Upsilon}{\partial t} \left| \frac{\partial \Upsilon}{\partial t} \right| \tag{2}$$

Considering μ as the drag constant and m_F as the fluid added mass, denoting $m = m_s + m_F$, we can rewrite equation (1) as:

$$m \frac{\partial^2 \Upsilon}{\partial t^2} = -EJ \frac{\partial^4 \Upsilon}{\partial z^4} + \frac{\partial}{\partial z}\left(T \frac{\partial \Upsilon}{\partial z}\right) - \mu \frac{\partial \Upsilon}{\partial t} \left| \frac{\partial \Upsilon}{\partial t} \right| \tag{3}$$

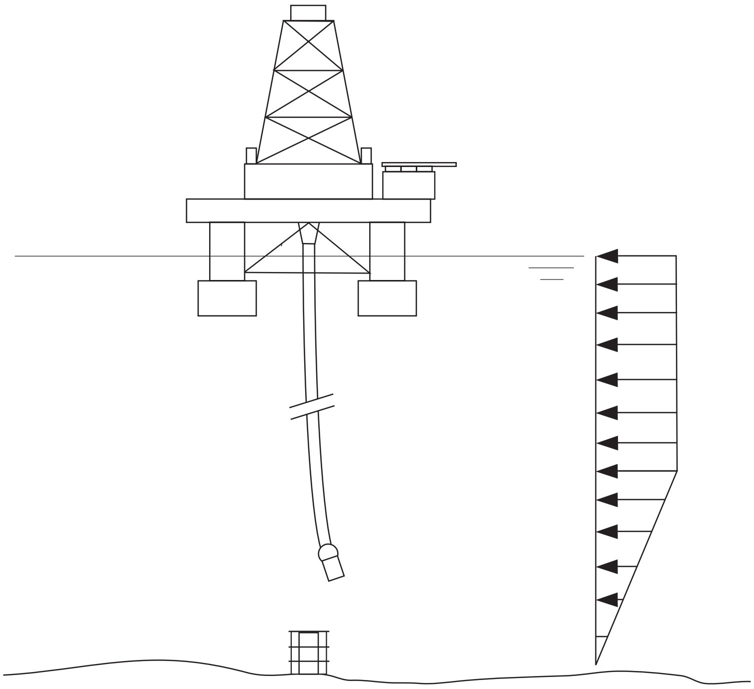

Figure 1: Platform during the reentry operation.

In practical cases, the displacement has low frequencies, so that $\frac{\partial}{\partial z}(T\frac{\partial \Upsilon}{\partial z}) \gg -EJ\frac{\partial^4 \Upsilon}{\partial z^4}$ (for low natural modes, the beam effects can be neglected). The tension for a disconnected riser in these conditions is a linear function of its weight ($T = (m_s - \rho S)z$), where ρ represents the water density and S the transverse section surface. Dividing equation (3) by m we get:

$$\frac{\partial^2 \Upsilon}{\partial t^2} = \frac{\partial}{\partial z}\left(\frac{(m_s - \rho S)z}{m}\frac{\partial \Upsilon}{\partial z}\right) - \frac{\mu}{m}\frac{\partial \Upsilon}{\partial t}\left|\frac{\partial \Upsilon}{\partial t}\right| \tag{4}$$

The constant term $(m_s - \rho S)/m$ can be replaced by an effective gravity g. It is proposed to linearize the drag term, substituting the term $\frac{\mu}{m}|\frac{\partial \Upsilon}{\partial t}|$ by the constant τ , that is calculated as a function of μ/m and of the mean value of $\frac{\partial \Upsilon}{\partial t}$ along the structure. With this approximation the system equation becomes the cable equation defined by Bernoulli (see [2]) plus a linear damping factor:

$$\frac{\partial^2 \Upsilon}{\partial t^2}(z,t) = \frac{\partial}{\partial z}\left(gz\frac{\partial \Upsilon}{\partial z}(z,t)\right) - \tau\frac{\partial \Upsilon}{\partial t}(z,t) \tag{5}$$

2 Motion planning

This section presents an analytical solution of equation (5) in the Laplace domain, following the idea proposed by Petit and Rouchon [2]. This solution is developed in Taylor's series, in order to formally inverse the Laplace transform. Then, it is possible to determine, in the time domain, an approximation of the open loop top riser trajectory, that is a function of the reference trajectory for the riser bottom. The first step is the variable change $l = 2\sqrt{z/g}$, which yields $\frac{\partial}{\partial z} = \frac{2}{gl}\frac{\partial}{\partial l}$, transforming equation (5) into a g-independent equation:

$$-l\frac{\partial^2 \Upsilon}{\partial t^2}(l,t) - \tau l\frac{\partial \Upsilon}{\partial t}(l,t) + \frac{\partial \Upsilon}{\partial l}(l,t) + l\frac{\partial^2 \Upsilon}{\partial l^2}(l,t) = 0 \tag{6}$$

Using a $t-$Laplace transform and considering the cable stopped at $t = 0$, the PDE (6) can be rewritten, with $\widehat{\Upsilon}$ the Laplace transform of Υ, as the following ODE:

$$-ls^2\widehat{\Upsilon}(l,s) - \tau ls\widehat{\Upsilon}(l,s) + \frac{\partial \widehat{\Upsilon}}{\partial l}(l,s) + l\frac{\partial^2 \widehat{\Upsilon}}{\partial l^2}(l,s) = 0 \tag{7}$$

The change of variable $\zeta = il\sqrt{s(s+\tau)}$ transforms (7) into a Bessel equation of the first kind:

$$\zeta\widehat{\Upsilon}(\zeta,s) + \frac{\partial \widehat{\Upsilon}}{\partial \zeta}(\zeta,s) + \zeta\frac{\partial^2 \widehat{\Upsilon}}{\partial \zeta^2}(\zeta,s) = 0 \tag{8}$$

The solution of $\widehat{\Upsilon}(z,s)$ has the following form:

$$\widehat{\Upsilon}(z,s) = c_1 J_0(2i\sqrt{s(s+\tau)}\sqrt{z/g}) + c_2 Y_0(2i\sqrt{s(s+\tau)}\sqrt{z/g}) \tag{9}$$

Here J_0 and Y_0 are respectively the Bessel functions of first and second kinds [7]. Sought after solutions are finite for $\zeta = 0$, so these solutions must be such that $c_2 = 0$:

$$\widehat{\Upsilon}(z,s) = \widehat{\Upsilon}(0,s) J_0(2i\sqrt{s(s+\tau)}\sqrt{z/g}) \tag{10}$$

Another way to define $\widehat{\Upsilon}(z,s)$ is:

$$\widehat{\Upsilon}(z,s) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta)\widehat{\Upsilon}(0,s)d\theta \tag{11}$$

There is no direct inverse Laplace transform for this expression that contains the arbitrary term $\widehat{\Upsilon}(0,s)$. The term $\exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta)$ in equation (11) can be expanded into a Taylor series around $\tau = 0$:

$$\begin{aligned} \exp(-2\sqrt{s(s+\tau)}\sqrt{\frac{z}{g}}\sin\theta) = \quad & \exp(-2\sqrt{s^2}\sqrt{z/g}\sin\theta) \\ & +\tau\left(\frac{\partial(\exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta))}{\partial\tau}\right)_{\tau=0} \\ & +\tau^2\left(\frac{\partial^2(\exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta))}{\partial\tau^2}\right)_{\tau=0} + \cdots \end{aligned} \tag{12}$$

The terms of the series can be rewritten in the following form:

$$\begin{aligned} \exp(-2\sqrt{s(s+\tau)}\sqrt{z/g}\sin\theta) = \quad & \exp(-2\sqrt{s^2}\sqrt{z/g}\sin\theta) \\ & \left(1-\tau\sqrt{\frac{z}{g}}\sin\theta + \tau^2\left(\frac{z\sin^2\theta}{g} + \sqrt{\frac{z}{g}}\frac{\sin\theta}{2s}\right)\right) + \cdots \end{aligned} \tag{13}$$

Using the Taylor series, the transfer function between the riser top and the bottom can be represented in the Laplace domain at the following form:

$$\begin{aligned} \widehat{\Upsilon}(z,s) = \quad & \frac{1}{2\pi}\int_{-\pi}^{\pi} \widehat{\Upsilon}(0,s)\exp(-2\sqrt{s^2}\sqrt{z/g}\sin\theta) \\ & \left(1-\tau\sqrt{\frac{z}{g}}\sin\theta + \tau^2\left(\frac{z\sin^2\theta}{g} + \sqrt{\frac{z}{g}}\frac{\sin\theta}{2s}\right)\right) + \cdots d\theta \end{aligned} \tag{14}$$

The interest of the Taylor series is to easily provide an inverse Laplace transform. The terms of the series can be analyzed as delays and easily associated to the reference trajectory $\Upsilon(0,t)$:

$$\begin{aligned} \Upsilon(z,t) = \quad & \frac{1}{2\pi}\int_{-\pi}^{\pi}\Bigg(\Upsilon(0,t-2\sqrt{\frac{z}{g}}\sin\theta)(1-\tau\sqrt{\frac{z}{g}}\sin\theta \\ & +\tau^2\frac{z\sin^2\theta}{g}) + \frac{\tau^2}{2}\int_0^t \Upsilon(0,\rho-2\sqrt{\frac{z}{g}}\sin\theta)\sqrt{\frac{z}{g}}\sin\theta d\rho + \cdots\Bigg)d\theta \end{aligned} \tag{15}$$

The open loop solution is found by numerically integrating equation (15), to define the control trajectory of the structure top $\Upsilon_o(L,t)$, where L is the riser length. The simulation example in Figure 2 shows that the approximations made have a small effect on the system response. The considered discrete system for all numerical simulations comes from the discretization (100 elements) of equation (1), for a vertical 2 km long steel riser with external diameter = 0.55 m and internal diameter = 0.5 m. These are typical values for a drilling riser in deep water. In figure 2, the hydrodynamic force is represented by equation (2).

3 System control

3.1 Disturbances

In practical cases, the structure has two main kinds of disturbances, that change the flow speed: waves and marine currents. The waves have their energy concentrated on the first meters of depth. They normally have two main frequencies, the faster around 50 mHz and the other around 10 mHz. The marine currents have their energy distributed more uniformly, with smooth time variations. The effect of the marine currents generates an offset that slowly changes. The changes of the flow speed due to these disturbances are represented by the function $U(z,t)$, that, with respect to the unperturbed case given by (2), induces the following changes in the hydrodynamic forces (m_I is a constant usually called the inertia coefficient):

$$F(z,t) = -m_F \frac{\partial^2 \Upsilon}{\partial t^2} + \mu \left(U(z,t) - \frac{\partial \Upsilon}{\partial t} \right) \left| U(z,t) - \frac{\partial \Upsilon}{\partial t} \right| + m_I \frac{\partial U}{\partial t} \tag{16}$$

3.2 Lyapunov control

Unperturbed case Consider $\Upsilon_R = \Upsilon - \Upsilon_o$ the relative displacement of Υ around the reference trajectory Υ_o. The objective is to define a tracking system to force the convergence of Υ_R to zero. Following the idea proposed by Thull et al [3], a candidate Lyapunov function, based on the system energy associated to Υ_R, is given by

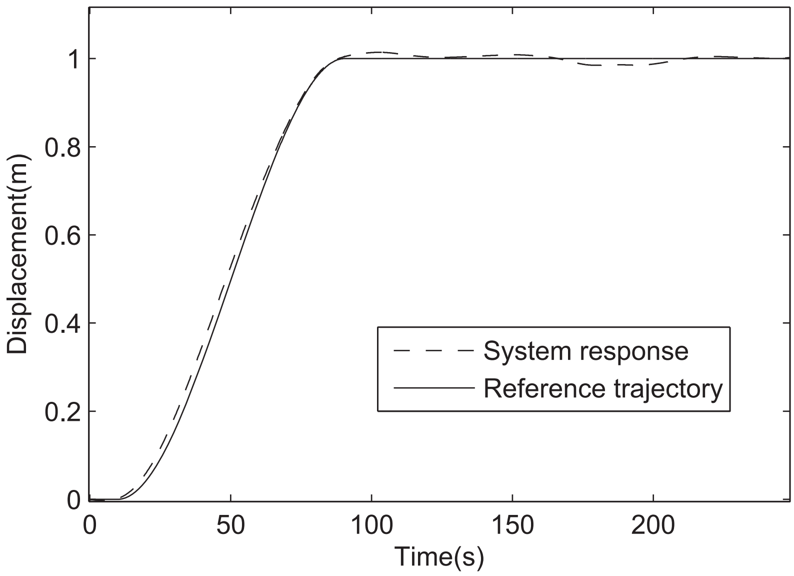

Figure 2: Unperturbed case. Reference trajectory and system response.

$$V = \frac{L\Upsilon_R^2(L,t)}{2\vartheta^2} + \frac{1}{2}\int_0^L \left(z\left(\frac{\partial \Upsilon_R}{\partial z}\right)^2 + \left(\frac{\partial \Upsilon_R}{\partial t}\right)^2 \right) dz \tag{17}$$

Parameter ϑ represents the convergence time and it is associated to the energy of the structure top relative displacement. Using equation (5), the time derivative of V is computed as follows:

$$\frac{dV}{dt} = \frac{L\Upsilon_R(L,t)}{\vartheta^2}\frac{\partial \Upsilon_R}{\partial t}(L,t) + \int_0^L \left(z\frac{\partial \Upsilon_R}{\partial z}\frac{\partial^2 \Upsilon_R}{\partial z \partial t} \right) dz + \int_0^L \left(\frac{\partial \Upsilon_R}{\partial t} \left(\frac{\partial}{\partial z}\left(gz\frac{\partial \Upsilon}{\partial z} \right) - \tau \frac{\partial \Upsilon}{\partial t} \right) \right) dz \tag{18}$$

After integration by parts, it comes

$$\frac{dV}{dt} = L\frac{\partial \Upsilon_R}{\partial t}(L,t)\left(g\frac{\partial \Upsilon_R}{\partial z}(L,t) + \frac{\Upsilon_R(L,t)}{\vartheta^2} \right) - \tau \int_0^L \left(\frac{\partial \Upsilon_R}{\partial t} \right)^2 dz \tag{19}$$

A proposed control law is

$$\frac{\partial \Upsilon_R}{\partial t}(L,t) = -\alpha \left(g\frac{\partial \Upsilon_R}{\partial z}(L,t) + \frac{\Upsilon_R(L,t)}{\vartheta^2} \right) \tag{20}$$

With this law, $dV/dt \leq 0$, so the system converges to the largest invariant set contained in $dV/dt = 0$. This set is such that

$$\int_0^L \left(\frac{\partial \Upsilon_R}{\partial t} \right)^2 dz = 0 \quad \text{and} \quad \frac{\partial}{\partial z}\left(gz\frac{\partial \Upsilon_R}{\partial z} \right) = 0 \tag{21}$$

This first relation implies $\partial \Upsilon_R/\partial t(z,t) = 0$ for all z and all t, so the solution of the second relation is $\Upsilon_R(z) = c\ln(z)$, where c is an arbitrary constant. At rest, the balance of the external forces is given, at the top, by the sum of F_t, accounting for the horizontal part of the tension at the top of the structure, and F_p, the resultant of the disturbances. By definition, F_t is proportional to $\partial \Upsilon_R/\partial z(L,t)$. In this unperturbed case, $F_p = 0$, so $F_t = 0$ and

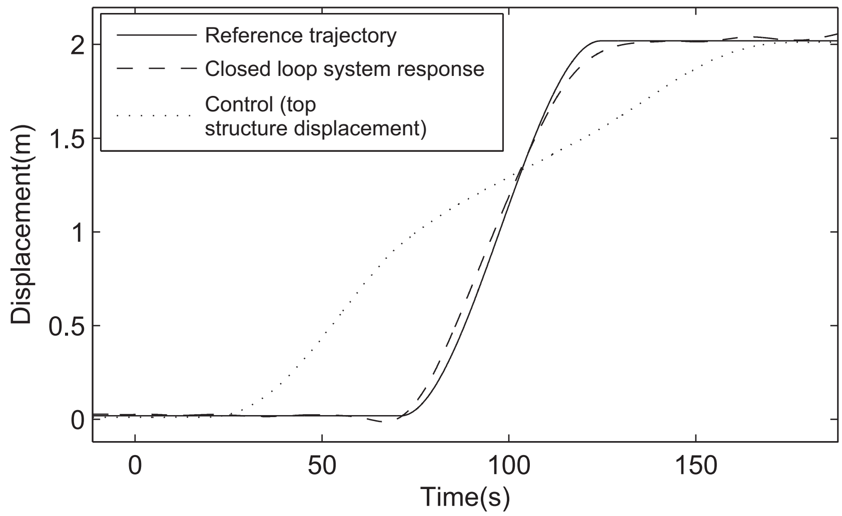

Figure 3: Lyapunov control in a case with constant disturbance due to the current. The control variable (top displacement) appears in dot lines

$\partial\Upsilon_R/\partial z(L,t) = 0$, which leads to $\Upsilon_R(L,t) = 0$. The unique possible solution for $\Upsilon_R(z) = c\ln(z)$ considering $\Upsilon_R(L) = (\partial\Upsilon_R)/(\partial z)(L) = 0$ is $c = 0$. That proves the convergence of Υ_R to zero for all z, and in particular gives $\Upsilon_R(0,t) = \Upsilon_R(L,t) = 0$: the re-entry operation is successful. Using the top position as the control variable, the control law writes in practice:

$$\Upsilon(L,t) = \Upsilon_o(L,t) - \alpha\int_0^t \left(g\frac{\partial\Upsilon_R}{\partial z}(L,v) + \frac{\Upsilon_R(L,v)}{\vartheta^2} \right) dv \tag{22}$$

Constant disturbances During the re-entry operation, the marine current can be assumed constant and generates a static deformation of the structure $\overline{\Upsilon}(z)$. If the assumption of small angles in the vertical direction holds, $\overline{\Upsilon}(z)$ can be defined by

$$-\frac{\partial}{\partial z}\left(gz\frac{\partial\overline{\Upsilon}}{\partial z}(z) \right) = \frac{\mu}{m}U(z)|U(z)| \tag{23}$$

with the boundary condition $\overline{\Upsilon}(L) = 0$. In these conditions, when the control law used in the unperturbed case is applied, it is easily proved that the system is stabilized at an equilibrium point given by $\Upsilon(z,t) = \overline{\Upsilon}(z) - g\vartheta^2\partial\overline{\Upsilon}/\partial z(L)$. To avoid this bias at the bottom, we propose the following solution. Before applying the control law:

- Estimate the current distance from the bottom to the wellhead. From this estimation, choose a reference trajectory for the bottom, and compute the top reference trajectory as in the unperturbed case.
- Estimate the current angle at the top ($\partial\overline{\Upsilon}/\partial z(L)$).

The definition of $\Upsilon_R(z,t)$ is modified into $\Upsilon_R(z,t) = \Upsilon(z,t) - \Upsilon_o(z,t) - \overline{\Upsilon}(z)$. Then, the computation of the control law obeys (22), exactly like in the unperturbed case. Indeed, the only difference lies in the use of the estimation $\partial\overline{\Upsilon}/\partial z(L)$. A typical closed loop simulation is presented in Figure 3. This simulation shows that good performances are obtained. In particular, there is no offset on the final position.

3.3 Feedback control based on system inversion

In the case of waves, the structure deformation linked to the disturbance is not constant. We still miss a formal approach for a Lyapunov function, that could guide us to a closed loop control in this case. For all the simulated situations, the control approach derived in the previous section does not provide good results in case of sea current plus waves. However there are two different alternatives to reduce disturbances effects. One alternative is to use the system inversion to define a tracking system, a second one is to choose an open loop trajectory, that reduces the disturbances influence on the system output.

The proposed approach consists in combining these two aspects using a tracking system with the inverted model defined by equation (15). Regarding equation (16), it is possible to observe that an artificial increase of the structure speed $\partial\Upsilon/\partial t$ implies a larger system damping, that reduces the relative effect of the flow speed changes. So, an open loop trajectory, that is fast enough to increase the damping during a given period of time, can reduce the effect of waves.

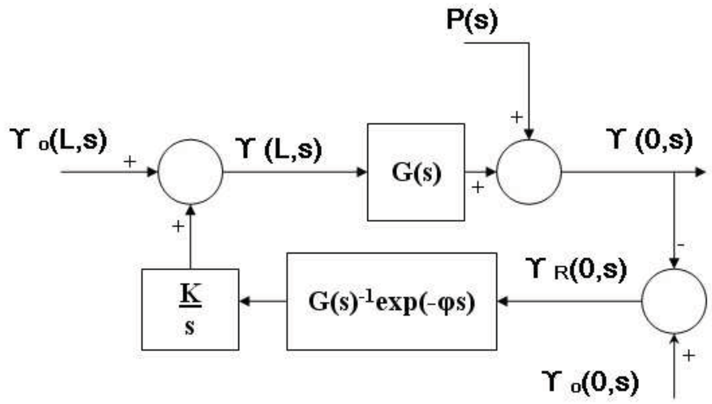

Figure 4: Block diagram of the tracking system.

Figure 4 presents the block diagram of the tracking system. The inverse of $G(s)$ is represented by equation (14). It is the modeled transfer function between the riser top $\Upsilon(L,t)$ and the riser bottom. Υ_o is the reference trajectory, Υ is the real riser position and Υ_R is the relative error between the riser position and the reference trajectory.

$G(s)$ has a maximum delay equal to $\phi = 2\sqrt{L/g}$ (see equation(15)), so its inverse $G(s)^{-1}$ should be associated to the same delay to become causal, in order to be used in the computation of a feedback control. The output of the block $G(s)^{-1}\exp(-\phi s)$ is the estimated control that would be necessary to avoid the relative bottom displacement $\Upsilon_R(0,t)$. The delay of the control estimation is an important problem for the high frequencies, however for the low frequencies this delay is negligible. The controller is an integrator with a gain K, so it ensures a static error equal to zero and has high gains associated to the low frequencies and low gains associated to the high frequencies, where the delay makes the estimate control useless. The advantage of this method is the possibility to compensate for the disturbance $P(s)$ not only due to the modeling errors, but also to the sea current and the waves.

The stability of this feedback law can be analyzed by the simplified Nyquist criterion. We can consider the open loop function $B(s)$ between $\Upsilon_R(0,s)$ and $\Upsilon(0,s)$, that writes $B(s) = K\exp(-\phi s)/s$. Rewriting $\exp(-\phi s)$ into its trigonometric form and replacing $s = i\omega$, we get:

$$B(s) = \frac{K(\cos(\phi\omega) - i\sin(\phi\omega))}{i\omega} \tag{24}$$

Considering $K > 0$, the most negative real value of equation (24) occurs for $\omega = \omega_0 = \pi/(2\phi)$. For this value we have $B(\omega_0) = -K/\omega_0$. A sufficient condition for the closed loop stability is $B(\omega_0) > -1$, which leads to $K < \pi/(2\phi)$. In practise, a smaller value of K is used, to provide robustness.

Figure 5 gives an example of what can be obtained with this approach for the control of a structure with waves and sea current disturbances. The use of the tracking system associated to the motion planning stabilizes the riser bottom at its target during a certain time (region close to $t = 1000$ s in the figure). During this time the connection of the riser bottom to the wellhead is possible. The shape of the reference trajectory is such that the reference speed is kept large almost until the end of the bottom displacement (but indeed this reference trajectory is chosen smooth enough to be followed on the nominal model with no modeling errors and no disturbances). This avoids the increase of the disturbances before the structure has reached its target. A problem with this trajectory is that, for a given disturbance, a maximum speed must be used, during a large enough period for the disturbances to be attenuated. This turns out to require a minimum initial distance between the bottom of the structure and the wellhead. The choice of this maximum speed is constrained by the following considerations, that are difficult to quantify a priori and do not preclude the constraints of the actuators:

- if the speed is too large, the small angles assumption does not hold, and more terms are required in the expansion of the damping term;
- if the acceleration is too large, the beam effect is no more negligible.

Conclusion

This article presents an analytical approximate solution for the trajectory planning of a damped cable. This solution is directly used to define the structure top displacement that is necessary for its bottom to follow a predefined trajectory. The convergence of the structure bottom to this trajectory is ensured by two different tracking systems. The first one is based on a Lyapunov function. It is able to compensate for constant disturbances due to the sea

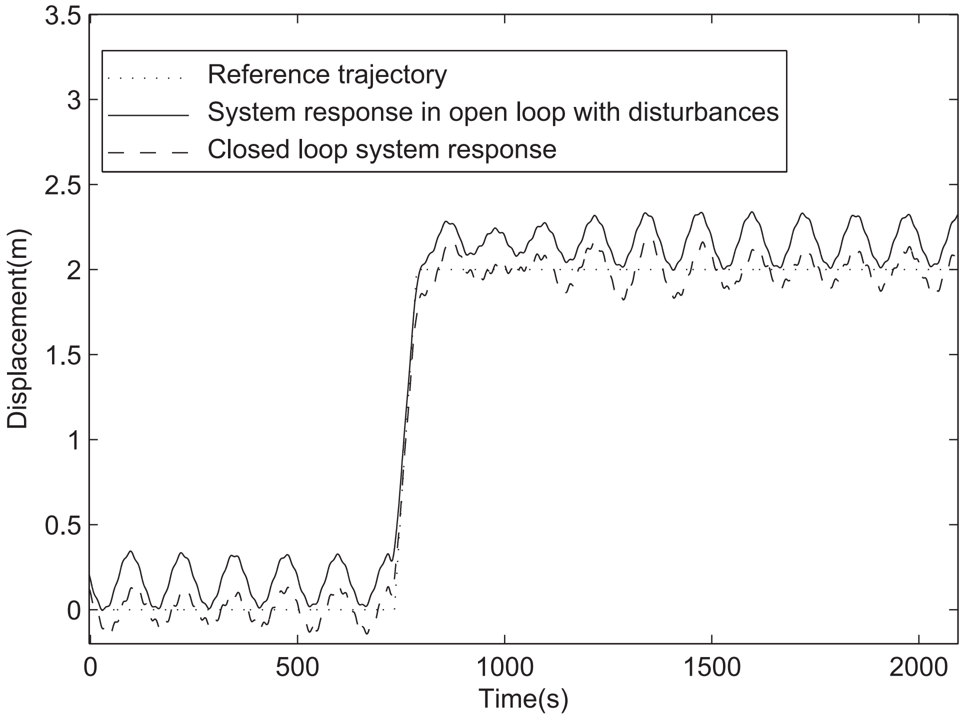

Figure 5: Structure under disturbances due to waves and sea current.

current. The second one is based on the model inversion. It is able to compensate for a larger group of disturbances, including waves.

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