Design of Nonlinear CMOS Circuits in the Nano-GHz Era and its Mathematical Challenges

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Abstract. Modern communication systems are working on industrial, scientific and medical (ISM) radio frequency (RF) bands. Today most research and development activities in industrial companies and universities are concentrated to frequency bands between 1 GHz and 24 GHz and even 60 GHz where semiconductor technologies are used for the construction of mixed-signal chips for cellular telephones and other wireless LAN applications (Bluethooth, HIPERLAN, etc.). In order to combine analogue functionality for the RF front end and digital functionality for high-speed signal processing nonlinear circuit concepts must be used for transceiver chips and CMOS technology has to be applied. In order to construct high-quality RF transceiver chips a submicron CMOS technology (180 nm and below) is needed. Therefore we come into the Nano-GHz Era of circuit design. In this talk corresponding mathematical challenges of modern RF CMOS design of transceiver circuits are discussed. It is shown that these circuits can be modelled as dynamical systems and its environment by partial differential equations. For circuit design aspects bifurcation theory and other mathematical concepts from dynamical systems implemented by a symbol manipulator (e. g. MAPLE, MATHEMATICA) as well as advanced numerical methods can be applied to build up an efficient design system for these RF circuits.

1 Introduction

During the last 25 years silicon CMOS became the main technology in microelectronics and following the IRTS roadmap for industrial applications CMOS will be the dominant technology in the next 15 to 20 years. One reason for the success of CMOS is related to the possibility that almost all circuit concepts survived the downscaling process from the μm to the sub-μm scale. In the meantime state-of-the-art technologies are below the 100 nm scale and enter the sub-100nm regime. Therefore we have to reconstruct all parts of the modelling process for CMOS devices. From a physical point of view new effects have to be studied in order to get reliable models for CMOS devices. Especially the charge transport is not only restricted to drift and diffusion processes but we have to consider also quantum mechanical aspects of transport. Since below 10nm gate length we have pure ballistic charge transport – so-called coherent transport – and above 100nm gate length drift and diffusion processes dominate – completely incoherent transport – a sub-100nm technology is characterized by partly coherent and incoherent transport. The main reason behind this changing is the interplay between the quantum mechanical tunneling process and different scattering processes such that the scaling parameter can be used to control the changing between fully coherent and fully incoherent transport behavior in CMOS devices. Obviously an deep understanding of these transport aspects is a fundamental assumption to develop new CMOS device models as well as studying the consequences to the behavior of sub-100nm circuits. In a subsequent section this problem will be discussed more detailed.

Also the functionality of today's microelectronic circuits used for technologies well above 100 nm must be analyzed in order to be sure that the desired circuit properties can be preserved also in the sub-100nm regime. Otherwise we need new circuit shapes for accurate modelling of nm semiconductor devices and its consequences for the properties of nm circuits become a challenging problem and close related to that interesting mathematical problems arise. Especially the above mentioned quantum effects and their possibly more involved circuit properties have to be considered very carefully.

However nano-scaling in CMOS technology is not only a subject of difficulties. Obviously a main advantage of each downscaled CMOS technology is that the devices density and therefore the complexity of circuits increases. However in high-speed digital CMOS circuits (e.g. memory circuits) downscaling is connected with the disturbance of the ideal circuit functionality and their performance decreases. Additional problems arise if more advanced semiconductor device concepts are considered. The problems of modelling high-speed circuits are discussed elsewhere (see [21], [55]) and for alternative concepts beyond CMOS see [27]. Although a similar tendency can be observed in nano-scaled analogue CMOS circuits the frequency behavior of RF CMOS devices will be improved because of the decreasing parasitic dynamic effects. If nano-scaled MOS devices are used in RF microelectronics we have the above mentioned advantages and disadvantages and therefore a trade-off is needed for a specific technology with its corresponding characteristic length. Furthermore the particular limits with respect to the circuit functionality of a certain technology are of special interest. Heterojunction bipolar transistors (HBTs) are superior to frequencies of 100 to 200 GHz [23] but deep sub-100nm CMOS technology seems to be useful for RF circuits for a few GHz since we can combine them with digital circuits. This approach is called system-

on-chip (SOC) concept. Therefore we have to consider modelling of nano-scaled CMOS transistors including RF modelling aspects such that we have Giga challenges in combination to the nano challenges.

In order to study the properties and the behavior of such nano RF circuits we need semiconductor device models which based on quantum mechanically coherent and incoherent charge transport. For studying RF circuits advanced MOS compact models are needed for circuit simulation. Most recently new MOS models of this kind are developed using a surface potential formulation; see chapters 2 and 3 in [22]. In contrast to the threshold or charge sheet formulations this formulation can be generalized in such a manner that quantum effects can be incorporated. Additionally in contrast to most intrinsic quasi-static quantum mechanically device models non-quasi-static effects have to be included; see [51]. Although advanced MOS compact models incorporate some quantum effect more elaborated models have to be developed for fully coherent transport; see [14] and our chapter 8 in [22]. In this paper we discuss different modelling approaches and their results with respect to the design of nano-scaled RF circuits as well as the corresponding mathematical challenges.

2 Electronic Circuit Design and Dynamical System Theory

Basically the design process of circuits and systems assumes that their properties – so-called specifications or specs – are prescribed and the design task is to construct a circuit or a system with these specs. From a mathematical point of view the design problem can be interpreted as an inverse problem. Cauer [10], [11] (see also Cauer, Mathis [12]) was the first who formulated the design of electrical filter circuits as an inverse problem ("inverse analysis") although this approach was already used long time before in the area of celestial mechanics; see [20]. Today inverse problems are in the scope of mathematics as well as signal processing and physics but until now this subject is rare in electrical engineering; see Chechurin et al. [13] for an exception. However inverse problems are the basis of engineering design problems because we start with the specs that characterize some aspects of the solutions of equations describing the circuit model and seek for these equations that corresponds to the circuit model. In the design of electrical filters and digital circuits inverse problems are called synthesis problems. Unfortunately inverse problems result in nonlinear mathematical problems even in the case of linear time-invariant circuits and systems. Maybe it will be seldom discussed because only very few inverse problems can be solved exactly.

In practical circuit design the corresponding inverse problem cannot be solved as a whole but it has to be decomposed into several steps where analysis and synthesis steps alternate. Therefore it is difficult for many mathematicians to follow the design approaches of electronic circuits and systems and only a few design problems are systematized in mathematical sense; see e. g. Allen [1] with respect to high frequency amplifier design. Practical circuit design is working in two steps: In a first design step the circuit architecture – *circuit shape* (O'Dell [45]) – has to be chosen whereas in the second step the free parameters of this circuit shape have to be determined. An experienced circuit designer knows which kind of circuit shape is needed for a desired circuit functionality. Since the descriptive equations or model equations of a circuit shape include free parameters it is not possible to work with a numerical circuit simulator where all parameters have to be determined. Therefore from a mathematical point of view we have a multi-parameter family of dynamical systems and therefore in a first step the qualitative behavior of such a family has to be studied. If we consider the parameter space of a certain family of systems we are interested in those points where the qualitative behavior is changing. The corresponding analysis can be done by methods from bifurcation theory of dynamical systems. Since most of the powerful methods are available only for one-parameter bifurcation problems the other parameters have to be determined by means of circuit theoretical and/or technological arguments.

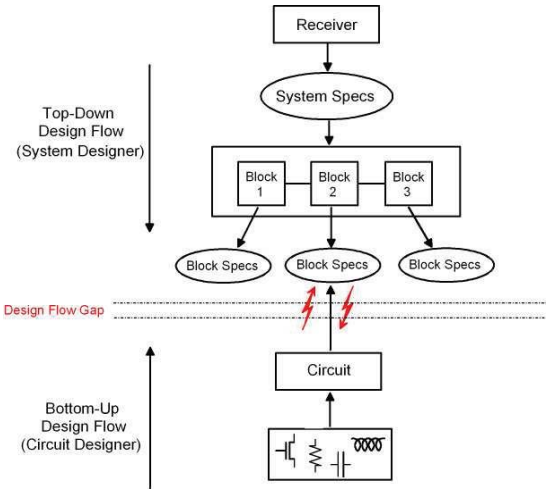

Figure 1: Gap between System and circuit design in current design flows

In the following we will discuss electronic circuits and systems design with applications in the GHz frequency

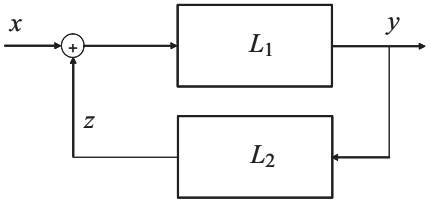

Figure 2: Feedback System

domain and under the influence of quantum effects. Therefore we consider only those circuit shapes and system architectures which are useful under these conditions. Most of these circuits and systems can be described by efficient mathematical models if nonlinearities will be included. There are two reasons for including nonlinear aspects into the models: 1. The functionality of circuits and systems based on nonlinear characteristics of semiconductor devices (e. g. CMOS transistors). 2. The Nonlinearities of the semiconductor devices influence the linear functionality of circuits and systems as disturbances. Obviously there is a close relationship between the specs of the functionality and the applied semiconductor technology. If a system designer develops the specs for a system it is not clear whether these specs can be realized by means of a certain semiconductor technology; this gap is shown in fig. 1. Many research work must be done in order to fill this gap in a practical manner; see e. g. [15].

Especially in the GHz frequency domain with its very fast changing of voltages and currents and if nanoelectronic technology is used designers have to be included both aspects into consideration. Although by means of nonlinear models we obtain a more accurate description of the devices it is more complicate to develop design processes. It is known from mathematics that almost all nonlinear equation have to be studied with special methods. Therefore automatic optimization concepts for analog CMOS design (e. g. Binkley [6]) arise but until now no successful implementation for general purpose analog design applications is available. Alternatively dedicated design algorithms for electronic circuits and systems can be developed based on well-adapted mathematical concepts for the nonlinear models. Most of these concepts for electronic circuits and systems including memory devices (e. g. capacitors and inductors) are available within the theory of mathematical dynamic systems. In this paper we will discuss some of these concepts with respect to their applications in circuit and systems design within the GHz-Nano era.

At first we will show that the descriptive equations of some modern system architectures just like phase-locked loop (PLL) systems and $\Sigma-\Delta$ modulators include memory terms and therefore we have a so-called non-Markovian descriptions. In mathematical terms these systems are described by integro-differential or integro-difference equations. The reason behind is that a model reduction process is applied to this kind of dynamical systems where the descriptive equations of a dynamic subsystem is solved. As a result this subsystem can be eliminated from the system model.

In order to illustrate that model reduction of certain dynamical systems leads to a non-Markovian model we consider a simple feedback system. It is described by

$$L_1(y) = x+z, \tag{1}$$
$$L_2(z) = y, \tag{2}$$

where x,y and z are time-dependent functions ($t \mapsto x(t)$ etc.) of a suitable function space, L_1 is an arbitrary (non)linear differential operator and L_2 a linear differential operator with constant coefficients (linear-time invariant). Since L_2 is a LTI operator we solve eq. (2) by means of

$$z = \int_R h(t-\tau)y(\tau)d\tau, \tag{3}$$

where h is the Green's function of L_2. Therefore we obtain the (non)linear integro-differential equation

$$L_1(y) = \int_R h(t-\tau)y(\tau)d\tau + x, \tag{4}$$

where the convolution integral corresponds the non-Markovian term. In next chapter we discuss some non-Markovian models in RF systems.

Additionally to the nonlinearities of semiconductor devices we have to study the functionality of circuits and systems under the influence of thermal noise and other kinds of noise. Since noise behavior of circuits and systems is a key property in the GHz-Nano era stochastic dynamic models instead of deterministic dynamic models have to be considered. A well-known physical concept to model noisy dynamical systems by means of a Langevin equation. This approach based on the deterministic equation where a white-noise stochastic process with a certain (constant) coupling factor is added. From a mathematical point of view a Langevin equation can be interpreted as a stochastic differential equation. Originally, the Langevin approach was developed for linear deterministic dynamic

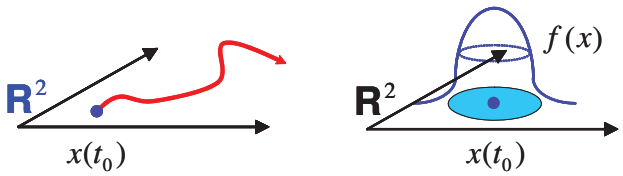

Figure 3: Types of Sets of Initial Points

models where a suitable physical interpretation exists. However in nonlinear deterministic dynamic systems some difficulties arise and alternative concepts have to be developed.

In the state space theory of deterministic dynamical systems the traditional approach emphasizes trajectories in the state space $\mathscr{S}$ where trajectories can be interpreted as solutions of differential equations with prescribed initial states. In certain cases the trajectory approach of dynamical systems is not useful. Boltzmann observed that the dynamics of sets of initial states instead of single initial state is suitable to calculate interesting physical quantities (see van Kampen [50], chapt. XIV). The deterministic behavior of a system is described by a set of differential equations

$$\frac{dx}{dt} = F(x), \tag{5}$$

where $F : \boldsymbol{R}^n \to \boldsymbol{R}^n$. If we are interested in the dynamics of a suitable class of density functions $f : \boldsymbol{R}^n \to \boldsymbol{R}$ an equation with an associated Frobenius-Perron-Operator P^t can be written down

$$f(x,t) = P^t\left(f(x)\right). \tag{6}$$

For an explicit representation we interpret the density function f as a probability function which fulfills the following conservation law

$$\frac{1}{N}\int f(x)dV = 1 \tag{7}$$

where dV is a suitable measure in the state space of the dynamical system with a suitable normalizing constant N. An equivalent formulation of this relation is the continuity law and together with the equations of motion (5) the generalized Liouville equation can be derived

$$\frac{\partial f}{\partial t} = -\mathrm{div}\left\{f \cdot F\right\} = -\sum_{i=1}^{n}\frac{\partial (f \cdot F_i)}{\partial x_i}. \tag{8}$$

This equation is an equivalent description of deterministic systems if we consider a whole set of initial conditions with a weighting function f instead of a single initial point x_0.

In the Langevin approach of stochastic systems we start with such a deterministic description and add a stochastic process ξ

$$\frac{dx}{dt} = F(x) + \sigma(x)\,\xi, \tag{9}$$

where the coefficient $\sigma(x)$ characterizes the coupling of the noise source. The first term should be interpreted as a dissipation term where as the second term corresponds to a fluctuation term.

Using the concept of stochastic differential equations ξ has to be interpreted as a generalized white noise process, but in order to solve these equations a more generalized concept of integration (e.g. Kunita [30]) is needed. In essential there are two concepts of stochastic integration which are due to Ito and Stratonovich, respectively, and associated types of stochastic differential equations

$$dx = F(x)dt + \sigma(x)\,dW, \tag{10}$$

where W is the so-called Wiener process. Both concepts are mathematically equivalent to a Fokker-Planck partial differential equation

$$\frac{\partial f}{\partial t} = -\sum_{i=1}^{n} \frac{\partial (f \cdot F_i)}{\partial x_i} + \sum_{i,j=1}^{n} \frac{\partial^2 (\sigma^2 f)}{\partial x_i \partial x_j}, \tag{11}$$

which generalizes in some sense the concept of the Liouville equation (8); see also Arnold [4], section 4.2. An alternative approach to the Fokker-Planck equation uses the stochastic Liouville equation and a suitable averaging process; see Kubo [29].

In the case of linear stochastic differential equations - the original subject of Langevin [31] - there is no difference between Ito's and Stratonovich's type. Unfortunately stochastic differential equations (of Ito or Stratonovich type) are sound concepts only from a mathematical point of view if we consider nonlinear Langevin equations. The reason is that each type corresponds to a certain interpretation rule; otherwise its meaning is not well defined. It is interesting to see that for nonlinear Langevin equations in contrast to linear ones the deterministic equation (without noise) does not correspond to the averaged equation (see van Kampen's paper for further details [48])

$$\left\langle \frac{dx}{dt} \right\rangle = \frac{d\langle x \rangle}{dt} = \langle F(x) \rangle + \langle \sigma(x)\, \xi \rangle \,. \tag{12}$$

Even if σ is constant and the stochastic part vanishes ($\langle \sigma\, \xi \rangle = \sigma \langle \xi \rangle = 0$) we note that the function F and the average operator $\langle \cdot \rangle$ do not commute. Only if $\langle F(x) \rangle = F(\langle x \rangle)$ is valid the averaged equation (first moment of the stochastic process x) fulfills the deterministic equation $\dot{x} = F(x)$. Therefore with van Kampen [49] we come to the conclusion that there is no good reason why the dissipation term should be identical to the vector field of the deterministic equation in nonlinear cases. In order to obtain a sound description of physical systems additional considerations are needed. Based on Stratonovich's ideas Weiss and Mathis ([52], [53] and [54]) have presented a substantial work for reciprocal electrical circuits.

Since integrated circuits contain semiconductor devices with a non-reciprocal and nonlinear model and therefore the Langevin approach can only used in an approximative manner without a sound physical reasoning.

3 PLL Systems and $\Sigma - \Delta$ Modulators

Probably the most important subsystems of modern RF transceiver systems are phase-locked loop (PLL) systems. As shown in fig. 4 the basic PLL structure consists of several blocks which are connected in a feedback structure. Additionally the input signal in fig. 4 is superimposed with bandpass noise. Although we have done some research with respect to noise sensitivity of PLLs (see Anders and Mathis [2]) we only restrict ourself to the deterministic case.

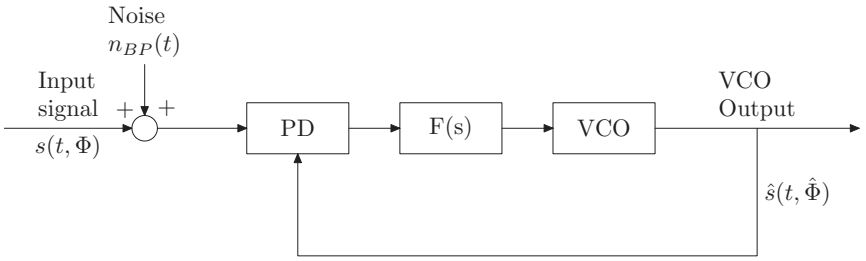

Figure 4: PLL configuration with bandpass noise process at the input

In order to derive the descriptive equations of a PLL system a mathematical model of the phase detector PD and the other blocks are needed. For this purpose a baseband model of analog PLLs can be developed where the system is described by means of phase signals. The phase detector is modelled by a static block with a sinusoidal characteristic whereas the filter block can be described in the frequency domain by the transfer function $F(s)$ and the voltage controlled oscillator (VCO) is simply modelled by an integrator block. In the time domain the filter block is working as convolution block. If the input phase is denoted by $\Theta(t)$ and the output phase of the VCO by $\tilde{\Theta}(t)$ a difference signal $\phi(t)$ can be defined that is the input signal into the PD model (sinusoidal block) which is filtered and will be the input signal of the VCO block. As the result we have the following integro-differential equation

$$\frac{d\phi(t)}{dt} = \frac{d\Theta(t)}{dt} - K_0 K_m K_1 A \int_{-\infty}^{+\infty} h_F(t-\tau) sin(\phi(\tau) d\tau \tag{13}$$

The convolution term can be interpreted in a physical sense as a non-Markovian property of a general PLL system, that is, a model reduction techniques is applied and the state space of the filter is not included explicitly in the description. Only if the filter block has a transfer function with $F(s) = 1$ the equ. (13) is reduced to a differential equation and standard techniques can be used for the analysis to build up a design concept for PLLs. Otherwise

more involved mathematical concepts are needed if the nonlinear behavior of PLLs take into consideration; e. g. studying the locking behavior.

Another interesting subsystem of modern CMOS signal processing systems are so-called ΣΔ modulators. Usual modelling techniques for ΣΔ modulators use linearized models in the frequency domain. A different approach was introduced in [43] by providing a time domain description and solving the characteristic nonlinear integro-difference equation, that is a non-Markovian model is developed. Finding a solution of this equation is complicated by the nonlinear comparator which can be described ideally by the Heaviside function. If certain conditions will be satisfied the argument of the Heaviside function can be replaced by the greatest whole number less or equal than the argument of the function.

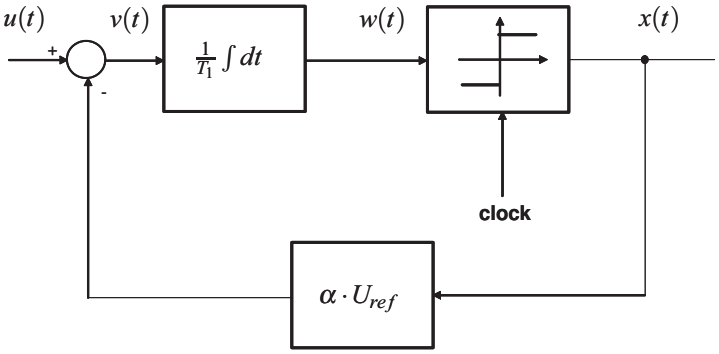

Figure 5: First Order CT ΣΔ modulator

The model illustrated in Figure 5 shows a first order ΣΔ modulator. According to the formula

$$x(t) = 2\sigma\{w(t)\} - 1 \tag{14}$$

the ideal comparator toggles its output $x(t)$ due to the sign of the input signal, where $\sigma\{.\}$ denotes the Heaviside function. The filter integrates the difference between the analog input signal $u(t)$ and the feed back output bitstream $x[k]$, weighted by αU_{ref}, resulting in the integrator output

$$w(t) = \frac{1}{T_I}\int_{t_0}^{t} u(\tau) - \alpha U_{ref} x(\tau) d\tau + w(t_0). \tag{15}$$

Combining the above equations and setting the initial condition $w(t_0) = 0$ yields to the following relation for the output bistream:

$$x(t) = 2\sigma\left\{\frac{1}{T_I}\int_{t_0}^{t} u(\tau)d\tau - \frac{\alpha U_{ref}}{T_I}\int_{t_0}^{t} x(\tau)d\tau\right\} - 1. \tag{16}$$

Due to the sampling inside the comparator the second integral can be replaced by a sum since the output bitstream is a discrete function $x[k] := x(k\Delta t)$, which is constant during a sampling period Δt. Thus (16) can be written as a nonlinear integro-difference equation

$$x[k] = 2\sigma\left\{\frac{1}{T_I}\int_{t_0}^{k\Delta t} u(\tau)d\tau - \frac{\alpha U_{ref}\Delta t}{T_I}\sum_{j=0}^{k-1} x[j]\right\} - 1. \tag{17}$$

Applying the definitions for the first difference

$$y[k+1] - y[k] := x[k], y[0] = 0 \tag{18}$$

the integro-difference equation can be solved explicitly. The solution is given by

$$y[k] = 2\left(\left\lfloor \frac{1}{2\alpha U_{ref}\Delta t}\int_{t_0}^{(k-1)\Delta t} u(\tau)d\tau \; + \frac{k-1}{2}\right\rfloor + 1\right) - k, \tag{19}$$

with $\forall k \in \mathbb{N}$ if the condition $|u(t)| \leq \alpha U_{ref}$ is satisfied and where $\lfloor \xi \rfloor$ denotes the next whole number less or equal than ξ.

Unfortunately this approach is with respect to the explicit solvability restricted to a first order modulator. Therefore methods are needed to study modulators with a higher dimensional state space what is related to the filter order.

In this chapter we haver shown that challenging nonlinear problems arise from the modelling process of PLLs and ΣΔ modulators where in general nonlinear integro-differential and integro-difference equations have to be solved. If we are interested in systematic design processes semi-analytical (approximative) solutions are needed. However the presented models are rather crude and more precise modelling is needed if modern semiconductor technologies will be applied. In the next chapters we consider VCOs and its systematic design a main building block of many RF architectures.

4 RF Oscillator Design

Although electronic oscillators belong to the very first electric circuits at the beginning of the twentieth century a complete systematic design concept for this class of electronic circuits is not available until now. One of the reasons is that the behavior of these dynamical circuits depends in an intrinsic manner on the nonlinearities of within the circuit and therefore we are confronted with nonlinear differential equations. The oscillatory circuit behavior is related from a mathematical point of view to the so-called limit cycles. However electronic oscillators are fascinating circuits in many sense because the progress in manufacturing technologies of electronic devices and circuits led to new challenges in oscillator modeling and new mathematical concepts for solving the descriptive equations of oscillators. In certain cases the behavior of an electronic oscillator should be influenced by the behavior of other electronic systems in a desired manner such that entrainment and synchronization effects arise. Therefore driven nonlinear oscillators and their descriptive equations have to be considered where even chaotic behavior can appear. From a physical point of view electronic oscillators can be interpreted as such systems where dissipative structures occur. Ilya Prigogine coined this phrase as a name for the patterns which self-organize in far-from-equilibrium dissipative systems and limit cycles are a special case of them; see Nicolis, Prigogine [44]. Such dissipative systems are nonlinear and have to be connected with a DC power supply for delivering energy into the system. Furthermore these systems interact with a heat bath where energy is dissipated; see fig. 6. Accordingly the fluctuations of the heat bath influence the oscillator as electronic noise. As a result electronic oscillators have to be modelled by driven nonlinear stochastic differential equations with limit cycle-type solutions. In most cases analytical solutions for this type of equations are not available and approximation concepts have to be developed. From this point of view electronic oscillator circuits are until now a source of inspiration for new mathematical and physical concept; see e. g. Guckenheimer [24].

However for the oscillator circuit design not only approximative solutions of certain descriptive equations are needed since at the first stages of a design process only very few circuit parameters are known. As mentioned above a circuit design concept consists of two steps where it is starting from the specifications of a circuit under design. These specifications are closely related to the solutions of the descriptive equations of the designed circuit. Therefore if not all circuit parameters are available it is even not possible to know whether the descriptive equations possess oscillatory or limit cycle solutions.

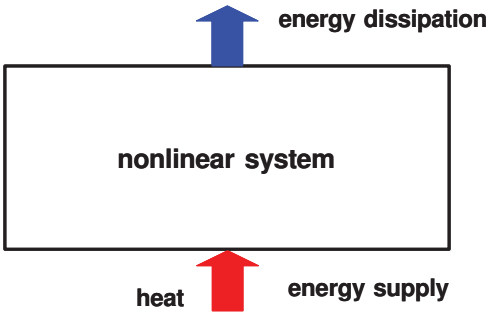

Figure 6: Dissipative Structure

For this reason Mandelstam and Papalexi [35] developed in 1931 the concept of parametrized descriptive equations for oscillator circuits based on ideas of the French mathematician Henry Poincaré. This concept was also the basis for the bifurcation theory of electronic oscillators; today it is called Andronov-Hopf theorem in the theory of dynamical systems. In contrast to the quantitative analysis of nonlinear differential equations this theorem studies these equations from a qualitative point of view. By means of the qualitative analysis we are able to consider the qualitative change of different types of solutions of nonlinear differential equations in dependence of certain circuit parameters. In the case of the Andronov-Hopf theorem the change from an equilibrium point to a limit cycle is explained. Although this theorem is known in electronic oscillator analysis since 1935 [36] (see Maggio et al. [34] for a more recent publication) it was never used for a systematic design process of electronic oscillators until recently; see Mathis & Russer [42], Prochaska et al. [47]. As mentioned above the circuit description with parametrized equations is well suited for the second step of the design process and therefore the Andronov-Hopf theorem should be used in design processes for electronic oscillator circuits; a first concept idea was presented by Mathis [39].

In the following sections of this paper we will discuss a systematic design approach of electronic oscillators developed in my research group where all steps of this approach are based on advanced mathematical tools from dynamical systems theory. Before we discuss the details of the design approach a sketch of the history of electronic oscillator theory is presented. Then we discuss deterministic and stochastic aspects of oscillator analysis useful for the oscillator circuit design.

5 Deterministic Aspects of Oscillator Design

5.1 The Andronov-Hopf Bifurcation

Although the descriptive equations of oscillatory circuits are in general of the type of the so-called differential algebraic equations (DAEs [37]) we will consider only those oscillator models which can be described by explicit ordinary differential or state-space equations

$$\dot{x} = F(x), \qquad F : \boldsymbol{R}^n \to \boldsymbol{R}^n \tag{20}$$

As already mentioned above we do not consider transient analysis problems but (stable) asymptotic solutions of (20) and especially limit cycles in oscillator circuits. This kind of solutions can exist only if the circuit is described by nonlinear differential equations. Note that in textbooks of circuit theory and design Barkhausen's criteria or alternative methods are used in order to determine the frequency and something like the "amplitude" of sinusoidal oscillators. However these methods are based on the linearized descriptive equations of oscillator circuits and therefore the question about the sense of these methods arises. The answer to this question was given by the author of this article in the year 2000 [39] by means of the Hartman-Grobman theorem. This theorem says that the dynamics of a nonlinear system in the neighborhood of an equilibrium point is equivalent to the dynamics of its associated linearized system only if it is a so-called *hyperbolic* system where the Jacobian matrix has only eigenvalues with nonzero real part; see also Mathis [40]. In this case the dynamical behavior is exponential. Only if we have a non-hyperbolic system where eigenvalues with zero real parts occur we can expect a more complex dynamical behavior - e.g. limit cycles. Since the Barkhausen-like criteria leads to a pair of eigenvalues on the imaginary axes a necessary condition for a stable limit cycles is fulfilled. This is the true reason behind the analysis of the associated linearized system in the sense of Barkhausen and others although the behavior of electronic oscillators is dominated by the nonlinearities.

In higher dimensions of the state space of a circuit no systematic methods for calculating limit cycles are available Papalexi, Mandelstam and Andronov developed the bifurcation approach for electrical oscillators using ideas from Poincaré's results. The main idea of these researchers was the embedding of (20) into a μ-parametrized family of differential equations

$$\dot{x} = F(x, \mu), \qquad F : \boldsymbol{R}^n \times \boldsymbol{R} \to \boldsymbol{R}^n \tag{21}$$

and searching for a qualitative changing of asymptotic solutions within this family.

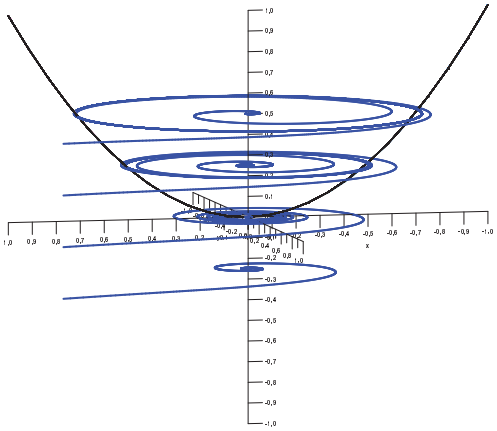

Figure 7: Andronov-Hopf Bifurcation

The Andronov-Hopf bifurcation theorem results in an efficient method for the analysis of limit cycles of autonomous one-parameter nonlinear ordinary differential equations of first order (21). By means of this theorem frequency and amplitude can be calculated in an approximative manner. Furthermore the stability of a limit cycle can be determined.

Andronov-Hopf Theorem:

If the following conditions

C1. the Jacobian matrix $J_F(x,\mu)|_{\mu=\mu_C}^{x=x_c}$ has a pair of conjugated complex eigenvalues $\lambda_{1,2} = \alpha(\mu) \pm j\omega(\mu)$ with the additional condition:

- $\omega(\mu)|_{\mu=\mu_c} = \omega_C > 0$,
- $\alpha(\mu)|_{\mu=\mu_c} = 0$,
- $d\alpha(\mu)/d\mu|_{\mu=\mu_C} \neq 0$

C2. all other eigenvalues have a negative real part

C3. and the first Lyapunov coefficient l_1 is negative, that is, the equilibrium point is asymptotic stable for $\mu = \mu_C$,

are fulfilled, then for bifurcation parameters $\mu > \mu_C$ a stable limit cycle exists and the oscillatory amplitude as well as the fundamental frequency can be approximated.

5.2 Bifurcation Analysis of a LC-tank VCO

In modern RF circuits for mobile communication systems so-called VCOs (Voltage Controlled Oscillators) for a few GHz belong to the most essential subcircuits. Therefore a design concept is needed where the necessary nonlinearity of VCOs has to be included. A first step to develop such a design concept based on the Andronov-Hopf theorem was the bifurcation analysis of LC-tank oscillators; see Prochaska, Belski, Mathis [47]. In order to design VCOs the voltage controlled capacitors have to be included in the analysis. Recently we present interesting semi-analytical results of a bifurcation analysis for a VCO in a 2.4 GHz CMOS technology where the charge-based EKV-MOS model is used; see fig. 8 and Bremer, Zorn, Mathis [7]. It turns out that the reduced descriptive equations have the form

$$\begin{pmatrix} \frac{di_L}{dt} \\ \frac{dv_t}{dt} \end{pmatrix} = \begin{pmatrix} 0 & -\omega_C \\ \omega_C & \alpha(\mu) \end{pmatrix} \begin{pmatrix} i_L \\ v_t \end{pmatrix} + \varepsilon f(v_t, i_L), \tag{22}$$

where

$$\alpha(\mu) = -\frac{1}{C_0(V_{tune})} \left[\frac{I_{bias}\sqrt{\mu_n C_{ox} W_n}}{2\sqrt{I_{bias} L_n}} - \frac{1}{R_t} \right], \tag{23}$$

and

$$\omega_C = \frac{1}{\sqrt{C_0(V_{tune}) L_t}}; \tag{24}$$

W_n is the width of the cross coupled pair and L_n is its channel length. $C_0(V_{tune})$ denotes the effective large signal capacitance of the VCO. It consists of the voltage dependant varactor capacitance $C_{v0}(V_{tune})$ and the parasitic capacitances of the other components of the VCO

$$C_0(V_{tune}) = C_{v0}(V_{tune}) + C_{par}. \tag{25}$$

Obviously we can show that the condition C.1 of the Andronov-Hopf bifurcation theorem is fulfilled if we have

$$W_{n,C} = \frac{4L_n}{\mu_n C_{ox} I_{bias} R_t^2} \tag{26}$$

and also the transversality condition C.1.3 and the stability condition are fulfilled. The limit cycle amplitude can be approximated by

$$r = \sqrt{\frac{8}{3}} \frac{\sqrt{R_t I_{bias}(R_t I_{bias} - v_n)} \cdot v_n}{R_t I_{bias}}, \tag{27}$$

where $v_n = \sqrt{(I_{bias} L_n)/(\mu C_{ox} W_n)}$.

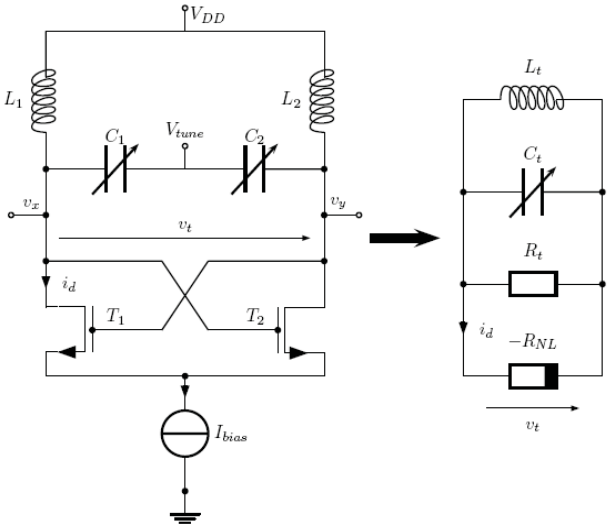

Figure 8: LC-tank Voltage Controlled Oscillator

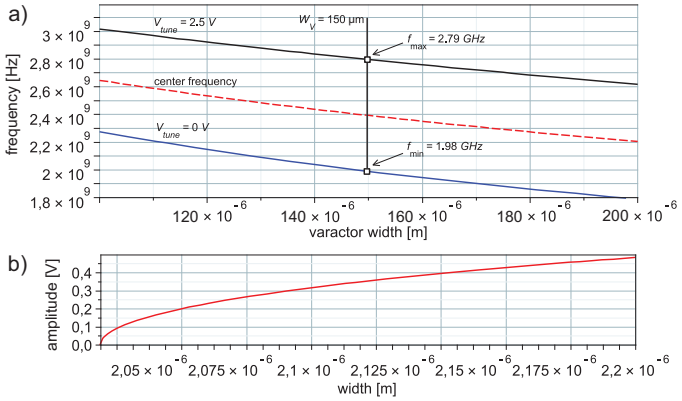

Figure 9: Maple: a) Estimated tuning range b) Estimated amplitude.

5.3 Example: Design of a 2.4 GHz LC-tank VCO

In this section we show the application of our proposed design concept to a practical oscillator design. The desired specifications are shown in Fig. 11 (a $0.25\mu m$ RF CMOS technology from IHP (SGB25V) is used). An inductor with a series inductance of $L_{i,s} = 11.1nH$ is chosen for the requested frequency range. Using our proposed model for the voltage dependent varactor capacitance [7] we are able to approximate the varactor dimensions. Figure 9 shows an approximation of the frequency characteristic of the VCO in dependency of the varactor width W_v. Our calculations show, that as a first estimation a varactor width of $W_n = 150\mu m$ is an appropriate choice for the requested frequency range (see Fig. 9).

Setting L_v to two times the minimum channel length is a good compromise between series resistance and $C_{v,max}/C_{v,min}$ ratio [8]. In order to minimize the parasitic resistance the length of the transistor pair L_n is set to the minimum channel length. We set the value of I_{bias} to the maximum value allowed by the specifications in order to maximize the output amplitude and optimize the phase noise characteristics of our VCO [25]. All design variables have been determined except the width W_n of the transistor pair. It is guaranteed that a variation of the width of the transistor pair W_n leads to a stable limit cycle as was shown in the previous section. Using expression (26) it is possible to calculate the needed width W_n. We find the bifurcation point and therefore the starting point of a stable oscillation at a parameter value of $W_n = 2.03\mu m$. Increasing W_n leads to an expansion of the limit cycle and accordingly to a rising of the oscillation amplitude (27) (see fig. 9).

The obtained design parameters are subsumed in Fig. 11. In order to validate the theoretical results we have simulated the VCO on circuit level in Cadence using a BSIM 3.4 MOS model. The simulation results show that a stable oscillation is built up at a width of the transistor pair of $W_n = 3.05\mu m$, which is pretty close to our calculated bifurcation parameter. Calculation of the tank amplitude is only valid close to the bifurcation point (see fig. 9) and here the amplitude is too small to fulfill the required specifications. Increasing W_n leads to a larger amplitude (see equation 27). We found that for a width of $W_n = 4\mu m$ the VCO has an amplitude of $v_t = 1.37V$ which fulfills the requested amplitude specifications. Increasing the width W_n beyond $4\mu m$ does not increase the amplitude anymore. Until now we have only ensured that the VCO possesses a stable oscillation for a tuning voltage of $V_{tune} = 0V$. An

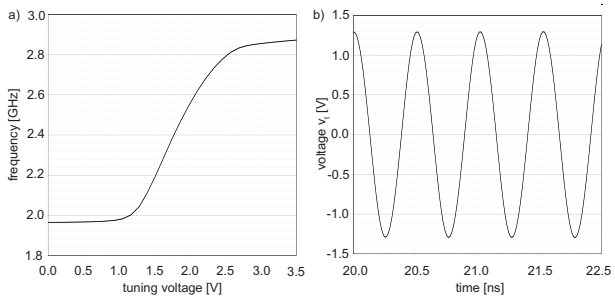

Figure 10: Cadence: a) Frequency characteristic b) Transient output signal.

Param.	Spec.	Achieved
f_0	2.4 GHz	2.4 GHz
V_{DD}	2.5 V	2.5 V
I_{bias}	≤ 1 mA	1 mA
TR	≥ 20 %	34 %
v_t	≥ 1 V	≥ 1.37 V
C_{load}	≥ 100 fF	100 fF

Inductor		
Param.	**Maple**	**Cadence**
$L_{i,p}$	11.16 nH	-
$C_{i,p}$	16.6 fF	-
$R_{i,p}$	1824 Ω	-

Transistor Pair		
Param.	**Maple**	**Cadence**
W_n	> 2.03 µm	> 3.05 µm
L_n	250 nm	250 nm
R_0	10 kΩ	9.84 kΩ
C_{NMOS}	3.97 fF	3.77 fF

Varactor		
Param.	**Maple**	**Cadence**
W_v	150 µm	140 µm
L_v	500 nm	500 nm
$R_{v,p}$	3222 Ω	-
C_{v0}	378 fF	-

Figure 11: Specifications and design parameters for $V_{tune} = 0$.

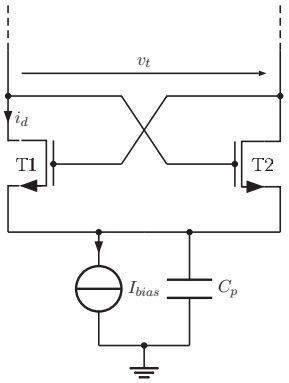

Figure 12: LC-tank VCO with filter capacitance

increase of V_{tune} leads to an decrease of the tank capacitance because of the voltage-dependent varactor capacitance and thus to a higher VCO frequency. An increase of the frequency leads to a higher R_t since the equivalent tank resistance R_t is frequency-dependant. According to (26) the required width W_n is inversely proportional to R_t^2, therefore it is ensured that the VCO designed at $V_{tune} = 0V$ oscillates for the whole tuning range. The proposed design concept enables to find the minimum W_n in order to built up a stable oscillation and additionally optimizes the amplitude of the VCO. Doing so the effect of the nonlinearity of the transistor pair and therefore the influence of the higher harmonics can be minimized. The design procedure leads to a VCO that possess a high spectral purity with a highly sinusoidal output signal.

5.4 Higher dimensional state space representation

A higher dimensional state space representation allows the inclusion of other parasitic effects and structural enhancements in comparison to the 2-dimensional state space modeling. Examples of possible parasitic effects could be the nonlinear effects of the cross-coupled transistor pair or substrate effects for instance. An example for a structural enhancement that could be included in the modeling using higher dimensional state space equations is a filtering capacitance parallel to the current source (see fig. 12). In order to carry out the bifurcation analysis of an sinusoidal oscillator with a higher dimensional state space in a more simple manner we have to calculate the corresponding center manifold and transform the descriptive equations to equations with a 2-dimensional state space. There are several important systematic techniques for the simplification of systems where simplification means that we can study the main aspects of a system on lower dimensional state space. The so-called center manifold reduction based on the center manifold theorem (e.g. Guckenheimer, Holmes [24]) takes advantage of the natural separation of timescales that occurs when some of the eigenvalues of a equilibrium point pass through zero. If a center manifold exists and no unstable solutions occur, the slow time dynamics dominate on this submanifold in the complete space and all other solutions rapidly approach this manifold; see fig. 13. To be more precise the vector field F of $\dot{x} = F(x)$ is divided into a linear and a nonlinear part

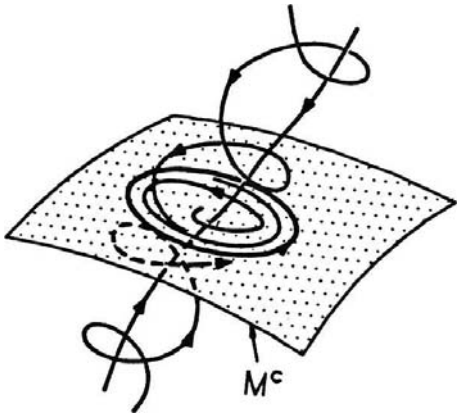

Figure 13: Transient Behavior near the Center Manifold

$$\dot{x} = Ax + F_c(x,y), \tag{28}$$
$$\dot{y} = By + F_s(x,y), \tag{29}$$

where the eigenvalues of A have only zero real parts and of B only negative real parts. The center manifold of this system is

$$W^c(0) := \{(x,y) \in \boldsymbol{R}^c \times \boldsymbol{R}^s | y = h(x), h(0) = 0, \tag{30}$$
$$Dh(0) = 0\}. \tag{31}$$

The function $h(x)$ is determined by

$$Bh(x) + F_s(x,h(x)) = Dh(x)(Ax + F_c(x,h(x))) \tag{32}$$

which is usually approximated in a power series. The simplified dynamics on the center manifold are described by the reduced system

$$\dot{z} = Az + F_c(z,h(z)) \quad u \in \boldsymbol{R}^c. \tag{33}$$

Moreover the center manifold approach for simplifying dynamical systems has a nice geometric interpretation.

Further investigations of the reduced system with analytical methods are well-known in system theory [24]. The following methods are predestinated for the use in common with center manifolds because they are formulated only in the 2-dimensional case or easier to handle in that dimension:

- *Poincare-Normal-Form:* A nonlinear system is transformed in a form showing the resonant (important) nonlinear terms [24]. The stability of equilibrium points can be estimated
- *Andronov-Hopf Bifurcation:* A system's steady-state can be changed by parameter from a stable equilibrium point to a stable limit-cycle [24] where oscillation conditions can be identified.
- *Amplitude-Angle-Form:* A system with periodic (limit cycle) solutions is transformed into new coordinates representing amplitude and angle of the expected solution [28] where a separation of amplitude and angle is used.
- *Averaged Form:* Periodic solutions are averaged over a period of the basic oscillation [24]. Via Lie-series transformation nonlinear oscillating systems can be averaged with the help of computer-algebra [37]. This corresponds to a filtering-out of high-frequency-oscillations.

The use of computer-algebra for the presented methods is of high value, because they all depend on equivalence transformations [40]. These coordinate transformations are well done by symbolic manipulators. The approach can be illustrated by means of rather simple oscillator circuits (e.g. Colpitts and Hartley oscillators).

5.5 Theoretical Aspects of Entrainment

There are several theoretical approaches to analyze the entrainment phenomenon where in general we have to consider a nonlinear oscillator with excitation; see e. g. Pikovsky et al. [46]. Because the associated nonlinear differential equations cannot be solved and a perturbation concept is needed (see e.g. Mathis [37]). Furthermore different kinds of differential equations with an oscillatory behavior can be presumed. A more simple method is Möller's "Schwingkennlinien" method (oscillatory characteristic; but see also Cassignol [9] and the generalized Barkhausen method) that is based on a first order Fourier series approximation. A reformulation of this method based on a feedback model of an oscillator where an excitation is incorporated. As a result the amplitude of an oscillator can be determined.

In more details an oscillator is modelled as a cascade of a nonlinear and a linear two-port where the output of the cascade is connected to its input; see e. g. Fack [17]. The nonlinear two-port is described by a nonlinear characteristic. The frequency characteristic of the linear two-port has a resonant form and only signals with the resonance frequency can pass. Therefore the corresponding term of the Fourier series of the output signal of the nonlinear two-port is necessary. In mathematical terms an input voltage of the nonlinear two-port is assumed

$$U_1(t) = Ee^{j(\omega t-\varphi)} \tag{34}$$

and at its output voltage can be expressed by

$$U_2(t) = Fe^{j(\omega t-\varphi-\psi)}. \tag{35}$$

From the feedback connection we have $U_1 = U_2$ or $E = F(E)$, where $F = F(E)$ is the nonlinear characteristic and the center frequency ω_0 of the passband of the resonator filter has to be introduced. It can be shown that $\psi = (\omega - \omega_0)t_0$. If a graphical representation is available the amplitude of the oscillator can be determined in a graphical manner using $E = F(E)$. In the case of entrainment an additional excitation voltage $U_e(t) = E_e e^{j\omega_e t}$ has to be included. Again a sinusoidal signal is assumed, but now E and φ are no longer constant but depend on time. In a similar way we derive the output voltage of the cascade of the two-ports

$$U_2(t) = F(E)e^{-t_0(\dot{E}/F(E))F'(E)}e^{j(\omega_e t-\varphi-(\omega_e-\omega_0-\dot{\varphi})t_0)}, \tag{36}$$

where the momentum frequency is determined in a more complicated way and ω_0 and t_0 are constants of the linear two-port. With feedback connection of the cascade $U_2 = U_1 - U_e$ we find

$$F(E)e^{-t_0(\dot{E}/F(E))F'(E)} = \sqrt{E^2 + E_e^2 - 2EE_e\cos\varphi}, \tag{37}$$

where

$$\tan((\omega_e - \omega_0 - \dot{\varphi})t_0) = \frac{E_e\sin\varphi}{E - E_e\cos\varphi}. \tag{38}$$

In the case of a horizontal part of the "Schwingkennlinie" we have $F'(E) \approx 0$ and therefore it results in

$$F(E) = \sqrt{E^2 + E_e^2 - 2EE_e\cos\varphi}. \tag{39}$$

If $E_e \ll E$ eq. (38) can be simplified to

$$\dot{\varphi}\,t_0 = (\omega_e - \omega_0)t_0 - \frac{E_e}{E}\sin\varphi. \tag{40}$$

Since this differential equation can be integrated, the following relation can be derived

$$\varphi(t) = \frac{\pi}{2} + \arccos\frac{m + f(t)}{1 + mf(t)}, \tag{41}$$

where $f(t) = \cos(\omega_e - \omega_0)\sqrt{1-m^2}t$ and $m = E_e/(E(\omega_e - \omega_0)t_0)$. We find out that φ will be constant if t approaches infinity for the case $m^2 > 1$; in this case $\sqrt{1-m^2}$ is complex. The value $\varphi(\infty)$ depends on m. If $m = 1$, we have $\varphi(\infty) = \pi/2$ and if $m = \infty$ it is $\varphi(\infty) = 0$. The cases of a constant phase are corresponding to the entrainment phenomenon. The phase will be increased if $m^2 < 1$ and $m = 0$ leads to a linear increasing behavior.

In conclusion we find out by this analysis that in the cases of a constant phase (entrainment) the phase varies between $\varphi = \pm\pi/2$ and $\varphi = 0$ where the concrete value of the asymptotic phase depends on m (that is circuit parameters).

6 Stochastic Aspects of Oscillator Design

In this chapter we consider two related concepts of general stochastic or noisy circuits and systems if a deterministic description of the form $\dot{x} = F(x)$ is assumed. If we consider non-reciprocal (nonlinear) circuits we have to apply the heuristic Langevin approach where an associated stochastic differential equation can be derived. From a system's theoretical point of view stochastic differential equations belong to the class of state space equations which are formulated in time-domain. An alternative concept of describing noisy circuits uses a probability density function f which satisfies a Fokker-Planck-type equation.

In this section we are concerned with parameterized families of stochastic dynamical systems in the Langevin form $\dot{x} = F(x,\mu) + G(x)\xi$ and its associated Fokker-Planck equation. Although it is known that both concepts are equivalent from a mathematical point of view, it turns out that there are different concepts of stochastic bifurcation. The earlier stochastic bifurcation concept is based on a Fokker-Planck description and was founded in physical applications by Horsthemke and Lefever [26]. In this approach qualitative changing of the stationary solution within the family of Fokker-Planck equations is studied. Although it is a suggestive concept which can be illustrated easily there is no time dependence and therefore it is rather a static concept to bifurcation. In the mathematical literature it is called "P-bifurcation" (e.g. Arnold [5]).

A dynamical concept of stochastic bifurcation is based on the stochastic differential equation. In contrast to the P-bifurcation concept where we look for qualitative changes of the asymptotic probability density function the dynamical (or D-) bifurcation concepts is concerned with qualitative changes of certain properties within the family of stochastic differential equations. For this purpose suitable analogues for equilibrium points of deterministic differential equations is needed. It turns out (see Arnold [5]) that so-called invariant measures of stochastic flows are adequate analogues for deterministic equilibrium points. In doing so we assume that like in the deterministic case a stochastic differential equation is replaced by a "stochastic flow" or so-called cocycle (Arnold [5]).

Note that if x_0 is a deterministic equilibrium point of a cocycle $\varphi(t,\omega,x_0) = x_0$ then the Dirac measure δ_{x_0} is stationary and invariant. Therefore there is a close relationship of deterministic equilibrium points and invariant measures.

Therefore the fundamental question of D-bifurcation is "Are there other invariant measures than Dirac measures?". It turns out that a necessary condition for qualitative changing in the sense of D-bifurcation is the vanishing of a Lyapunov exponent. It should be mentioned that there is no general relation between P-bifurcation and D-bifurcation.

For illustrating these bifurcation concepts we restrict us to 2-dimensional systems. For higher dimensional systems stochastic concepts for normal forms and/or center manifolds are needed (see Arnold [5]). We consider a Meissner oscillator circuit in fig. 1. If $k_2 = 0$ the following circuit equation for the voltage between base and emitter can be derived ($\omega_0^2 = 1/(LC)$)

$$\ddot{u}_{BE} + \left(\frac{R}{L} - \omega_0^2 M(k_1 - 3k_3 u_{BE}^2)\right)\dot{u}_{BE} + \omega_0^2 u_{BE} = 0. \tag{42}$$

Equation (42) can be normalized in the standard van der Pol form $\ddot{x} - (\mu - \gamma x^2)\dot{x} + x = 0$. Now we assume with Ariaratnam [3] that we have a noisy resistor which results in an additive decomposition of

$$\ddot{x} - (\mu_0 + \sigma\xi - \gamma x^2)\dot{x} + x = 0. \tag{43}$$

If (43) is converted into first order equations and polar coordinate transformations are applied to the system, we obtain after a stochastic averaging the following stochastic differential equation for the amplitude process $a(t)$

$$da = \frac{1}{2}\left(\mu_0 + \frac{5}{8}\sigma^2 - \frac{1}{4}\gamma a^2\right)a\,dt + \left(\frac{3}{8}\right)^{\frac{1}{2}} a\,dW_a. \tag{44}$$

For the analysis of D-bifurcations we have to determine the stability of stationary solutions $a_s(t)$ by means of associated Lyapunov exponents. If small variations $r(t)$ of $a_s(t)$ are considered the following linearized stochastic differential equation for amplitude process can derived

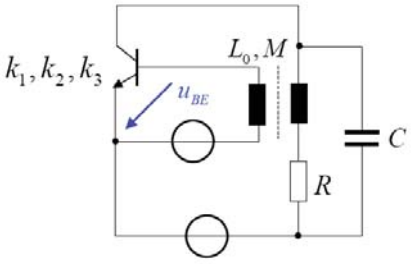

Figure 14: Transistor Meissner Oscillator

$$dr = \frac{1}{2}\left(\mu_0 + \frac{5}{8}\sigma^2 - \frac{3}{4}\gamma a_s^2\right) r dt + \left(\frac{3}{8}\right)^{\frac{1}{2}} \sigma r \, dW_a. \tag{45}$$

For the bifurcation analysis the zeros of the Lyapunov exponents have to be found

$$\lambda = \frac{1}{2}\left(\mu_0 + \frac{1}{4}\sigma^2 - \frac{3}{4}\gamma\langle a_s^2\rangle\right). \tag{46}$$

Obviously λ is zero for the trivial solution $a_s = 0$ under the condition $\mu_0 = -\sigma^2/4$. Furthermore for the same value of μ_0 we have a zero for $a_s \neq 0$, such that we have a D-bifurcation.

For studying the P-bifurcation we need a solution of the stationary Fokker-Planck equation associated to the noisy van der Pol equation. It turns out that a first P-bifurcation occurs at $\mu_0 = \sigma^2/8$ where the peak of the probability density function shifts as $a_p := 2((\mu_0 - \sigma^2/8)/\gamma)^{1/2}$. Another changing occurs at $\mu_0 = \sigma^2/2$ where the uni-modal density centered at the origin changes to a bi-modal density possessing a ring of peaks; see Ariaratnam [3]. Note that the μ_0-values for the D- and P-bifurcation differ substantially. Additional examples can be found in Arnold [5].

7 Quantum Effects in Nanoelectronic Circuits

Until now we considered some mathematical challenges with respect to nonlinearities and noise in circuits and systems. These aspects are already known since a long time in circuit and system analysis but additional questions arise if we consider the development of design concepts of circuit and system. For this purpose we have to formulate descriptive equations with free parameters to adapt the properties of a circuit shape or system architecture to the prescribed specifications of the design. In the GHz-Nano era not only nonlinearities and noise are difficult subjects in circuit and system design but in deep sub-100*nm* technologies extreme thin gate oxide of CMOS transistors leads to tunnel leakage currents and also tunnel currents can flow through the source to drain potential barrier. Furthermore band-to-band tunneling as well as charge confinement and quantization effects arise.

In the following we restrict ourself to CMOS device where several approaches to study above mentioned quantum effects are available. The main groups are full quantum mechanical models (e. g. nonequilibrium Green's function method) as well as modified versions of the classical drift-diffusion method and the so-called hydrodynamic transport models. Further details can be found in the monograph of Ferry and Goodnick [19] and results of our group in Mathis et al. [41] and Felgenhauer er al. [18]. One of the most interesting aspects is the changing in the physical interpretation of charge transport within the nano regime. Whereas the classical and quasi-classical transport models (e. g. Drude's model) based on electromagnetic field theory including dissipative effects it was observed by Landauer in 1957 [33] that a new quantum mechanical approach is necessary to understand the charge transport in the mesoscopic (and nano) regime. A main problem of a quantum mechanical description is that dissipation which seems to be an intrinsic aspect of electric conductance cannot described such that an alternative model for this macroscopic observable effect is needed. A first step was the Landauer model where conductance was described as an elastic scattering process with transmission and reflection. As a result Landauer obtained his famous formula for two terminal conductance

$$G = \frac{2e^2}{h} T, \tag{47}$$

where T is the transmission probability. Of course, Landauer's concept considers a ballistic conductor where we expect that in the case without scattering, that is $T = 1$, the resistance is zero. Landauer's formula results in $G = 2e^2/h \approx 12.9\,k\Omega$.

Similar as dissipation we cannot describe inelastic scattering effects in quantum mechanics such that an alternative modelling approach is needed. Büttiker was the first who coupled an electron reservoir by a lead to a mesoscopic system as an inelastic scatterer. In quantum mechanics we interpreted it as a quantum phase breaking process and it is denoted as decoherence. In this sense the ballistic transport is called coherent whereas the classical transport is called incoherent transport. Charge transport in MOS transistors within the sub-100*nm* regime is something in between and therefore we called it partly coherent transport. Consequently we have two possibilities to describe partly coherent charge transport: 1. A quasi-classical drift-diffusion model is used and it is modified by quantum mechanical aspects. 2. A ballistic model is used and scattering aspects are included. In our research we applied the latter approach based on the so-called nonequilibrium Green's function (NEGF) method.

In order to understand the basic idea of the NEGF for analyzing ballistic transport a very simple 1-dimensional device model is considered; see fig. 15. A more detailed description can be found in the monograph of Datta [16].

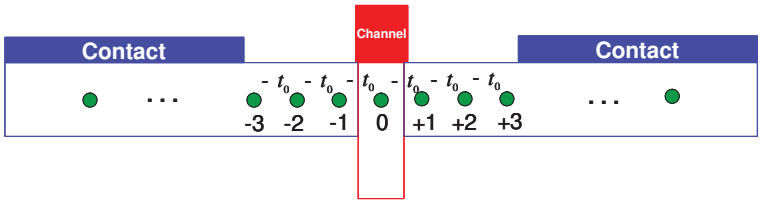

Figure 15: 1-dimensional Quantum Channel

The device consists of a charge transport channel together with a contact on each side of the channel. Each contact consists of equidistant arranged discrete mass points – with the distance a – interacting with its neighborhoods and a single channel mass. The crucial point of the behavior of this model is the contact-channel border crossing. Therefore the Schrödinger equations for this many-particle system have to formulate but we are only interested in the behavior of the channel mass. At the border between contact and channel a boundary conditions – continuity – is needed. The Schrödinger equation of the wave function for the interacting nth mass can be formulated in the following manner

$$E\Phi_n = -t_0\Phi_{n-1} + (2t_0 + E_c)\Phi_n - t_0\Phi n + 1, \tag{48}$$

where $t_0 := \hbar^2/(2ma)$. These equations can be solved as follows

$$\Phi_n = Be^{+ikna} + Ce^{-ikna}, \tag{49}$$

where $E = E_c + 2t_0(1-\cos ka)$ is the dispersion relation. The channel mass is described by

$$E\Psi = (2t_0 + E_c)\Psi \tag{50}$$

and the boundary condition is $\Psi = \Phi_0 = B + C$. Altogether we obtain

$$E\Psi = (2t_0 + E_c)\Psi + (-t_0 e^{+ika}\psi) + t_0 B\left(e^{+ika} - e^{-ika}\right). \tag{51}$$

If we define $S := it_0 B\sin(+ka)$ and $\Sigma := -t_0 e^{+ika}$ equ. (51) can be reformulated by

$$E\Psi = H\Psi + \Sigma\Psi + S, \tag{52}$$

where H is the Hamilton operator and Σ is the "self-energy" of the contact. As a result a inhomogeneous Schrödinger equation is obtained

$$\{(H+\Sigma) - E1\}\Psi = -S \tag{53}$$

that describes the open quantum system contact-channel. For solving equ. (53) the Green's function approach can be applied. For this purpose the solution G of

$$\{(H+\Sigma) - E\mathbf{1}\}G = -\delta, \tag{54}$$

where $\mathbf{1}$ is the operator unity and the solution of equ. (53) is given as $\Psi = GS$. The Green's function of the contact-channel model is

$$G(E) = \frac{1}{E - (E_c + 2t_0 - 2t_0 e^{ka}} = \frac{1}{2it_0\sin ka}, \tag{55}$$

the dispersion relation is used for last step. The density of state per unit cell is given by

$$D(E) = \frac{i}{2\pi}(G - G^+) = \frac{1}{2\pi t_0\sin ka} = \frac{a}{\pi dE/dk}. \tag{56}$$

The general description of a contact-channel device consists of a contact (reservoir R) interacting with the channel, that is

$$\begin{pmatrix} E\mathbf{1}-H_R+i\eta & -\tau^+ \\ \tau & E\mathbf{1}-H \end{pmatrix}\begin{pmatrix} \Phi_R+\xi \\ \Psi \end{pmatrix}=\begin{pmatrix} S_R \\ 0 \end{pmatrix}. \tag{57}$$

From this matrix equation the descriptive equation for the channel can be extracted

$$[E\mathbf{1}-H\Sigma]\Psi=S, \tag{58}$$

where $\xi = G_R\tau^+\Psi$, $G_R := [E\mathbf{1}_R - H_R + i\eta]^{-1}$ as well as $\Sigma := \tau G_R\tau^+$ and $S := \tau\Phi_R$. S is the "source" term representing the excitation of the channel by electron waves from the contact and Σ is the self-energy matrix that can be interpreted as the modification of Hamiltonian H to incorporate the "boundary conditions" (periodic boundary conditions). It is easy to add the other contact and we obtain the description for the arrangement contact 1 - channel - contact 2

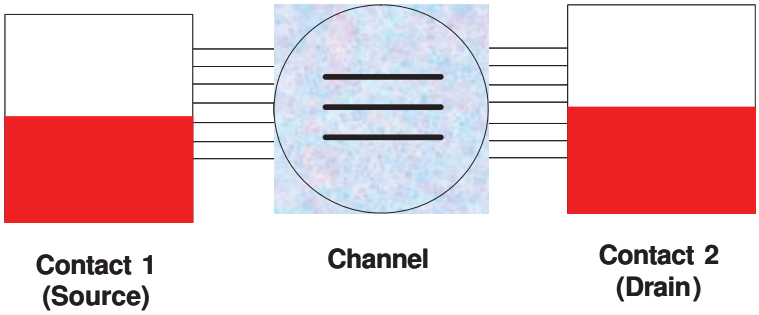

Figure 16: Contact 1 – Quantum Channel – Contact 2

$$\begin{pmatrix} E\mathbf{1}-H_1+i\eta & -\tau_1^+ & 0 \\ -\tau_1 & E\mathbf{1}-H & -\tau_2 \\ 0 & -\tau_2^+ & E\mathbf{1}-H_2+i\eta \end{pmatrix}\begin{pmatrix} \Phi_1+\xi_1 \\ \Psi \\ \Phi_2+\xi_2 \end{pmatrix}=\begin{pmatrix} S_1 \\ 0 \\ S_2 \end{pmatrix} \tag{59}$$

and in the same manner the contact reservoirs can be eliminated and the Schrödinger equation for the channel can be derived

$$[E\mathbf{1}-H\Sigma_1-\Sigma_2]\Psi=S_1+S_2, \tag{60}$$

where $\Sigma_i := \tau_i G_i \tau_i^+$ and $S_i := \tau_i\Phi_i$ for $i = 1,2$.

If we analyze the 1-dimensional contact-channel-contact example and connect the contact with voltage U_0 we have

$$\Sigma_{1,2} = -t_0 e^{+ika},\ quadH = [E_c + 2t_0 + (U_0/a)] \tag{61}$$

and

$$G = [E\mathbf{1}-H-\Sigma_1-\sigma_2]^{-1} = [E-E_c+2t_02t_0e^{+ika}-(U_0/a)]^{-1} = [2it_0\sin ka-(U_0/a)]^{-1} = \left[\frac{a}{iha\nu-U_0}\right]. \tag{62}$$

If we define the following quantities $[\Gamma_{1,2}] = i[\Sigma_{1,2}-\Sigma_{1,2}^+] = 2t_0\sin ka = ha\nu/a$ the transmission probability in the sense of Landauer can be calculated

$$T(E) = Tr[\Gamma_G\Gamma_2G^+] = \frac{\hbar^2\nu(E)^2}{\hbar^2\nu(E)^2+U_0r}. \tag{63}$$

Based in the Green's function approach and the assumption that we consider only stationary charge transport processes is considered where the grid atom are fixed, the crystal grid is time-invariant and the Fermi particle character is omitted we can solve the many-particle problem by means of a quasi-one-particle problem and the electron and current densities derived.

Based on these calculations the quasi-classical lumped model for MOS transistors (e. g. BSIM3 or EKV model) can be modified such that quantum effects can be incorporated. For this purpose circuit elements are defined by look-up table models or by means of elements with nonlinear characteristics where the corresponding parameters are determined by the simulation results of the above mentioned quantum calculations. Of special interest for the development of design algorithms are all-region semi-analytical MOS models. Therefore the approximative EKV-MOS model is very useful (see [22] for an overview) with the following description of the drain current I which depends of the gate as well as the drain and the source potential

$$I(V_G,V_D,V_S) = \frac{W}{L}I_s\ \log^2\left(1+e^{(\kappa(V_G-V_{TO})-V_S)/2U_T}\right) - \frac{W}{L}I_s\ \log^2\left(1+e^{(\kappa(V_G-V_{TO})-V_D)/2U_T}\right). \tag{64}$$

If we are interested in the weak inversion mode we have to approximate within the current formula $\log^2(1+\exp(x/2)) \approx \exp(x)$ (for $x<0$) and by $\log^2(1+\exp(x/2)) \approx (x/2)^2$ (for $x>0$) if we want to work in the strong inversion mode. The resulting current formulas are the well-known classical formulas for these working modes. A main advantage is that the EKV-MOS model interpret the drain current as the difference of a forward current I_F and a backward current I_R. In the weak inversion mode the diffusion current in the channel is dominant whereas the drift current is dominant in the strong inversion mode. A threshold voltage is not necessary in the EKV model but at a certain gate voltage the drift current and the diffusion current is equal and in this manner a characteristic voltage is defined by a clear physical mechanism.

Based on the above described quantum charge transport concept associated lumped MOS models can be derived and the functionality of integrated circuits can be studied under the influence of quantum effects. Further details of our research can be found in Felgenhauer [18]. Moreover using the all-region MOS model a semi-analytical MOS-varactor model was developed and applied for the design process of LC-tank oscillators.

8 Conclusion

In this paper we considered the essential steps of a systematic concept for the design of nonlinear and noisy electronic circuits and especially electronic oscillators. It can be shown that by means of advanced mathematical methods the intrinsic nonlinear problem of oscillator design is manageable. Using a high-efficient computer algebra system (e. g. MAPLE or MATHEMATICA) semi-analytical expressions for the oscillator design can be derived. Based on our previous work further research will be done in order to develop a computer-aided design for electronic oscillators in the GHz area. Furthermore an advanced concept for modelling quantum effects in MOS devices is presented and its relationship to circuit analysis is discussed. It is our goal to enable expert analog designers to design oscillators by means of a variety of well-adapted tools instead of a general purpose circuit simulator. In conclusion it can be emphasize for the development of design concepts for modern integrated circuits constructed for the GHz-Nano era a rather complicate interplay between deep physical modelling concepts and advanced mathematical concepts are are essential in order to obtain high efficient design processes useful for practical working circuit designers.

9 Acknowledgment

The authors would like to thank all his (former) assistants and (former) students who are working in my group since 1985 on analog circuit design and especially oscillator and synchronization aspects. Especially we thank Dr.-Ing. L. Weiss (Qimonda), Dr.-Ing. F. Freund (Bosch, Reutlingen), Dr.-Ing. Marcus Prochaska (NXP Hamburg), Dipl.-Ing. K. Bohle (Cameron GmbH, Celle), Dipl.-Ing. A. Belski (Leibniz Universität Hannover), Dipl.-Ing. Judith Schmackers (Bergische Universität Wuppertal), Dipl.-Ing. J. Anders (EPFL Lausanne, Switzerland) as well as Dipl.-Ing. A. Darrat, Dipl.-Ing. B. Fei as well as can. el. Chr. Zorn, cand. el. Tina Thiessen, cand. el. J. Przytarski (all Leibniz Universität Hannover) for their engaged working in our RF analog circuit design team.

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