

# PATTERN FORMATION AS A TRANSIENT PHENOMENON IN THE NONLINEAR DYNAMICS OF A MULTI-AGENT SYSTEM

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**Abstract.** This paper presents a microscopic model (agent positions, directions, and interactions are explicitly modeled) of mobile agents (or self-propelled particles) that is inspired by the “complex transport networks” reported by [4]. Here, the agents’ positions are modeled continuously. This multi-agent system (or artificial swarm) shows a wide variety of self-organized pattern formations. The self-organization is based on the nonlinearity of the agents’ turns (discrete jumps in the agents’ directions) and the indirect interactions of the agents via a potential field that determines their motion (high values are attractive) and which is changed by themselves (agents increase the value of the potential field at their positions). At least most of the irregular and complex patterns are transient. The patterns found during the transient are more complex than those the system converges to. Still, this transient behavior is relevant. I empirically investigate the transient times in dependence of the system’s size and give examples.

## 1 Introduction

The system, that is investigated in this paper, is inspired by and similar to the “complex transport networks” reported by [4]. In the system reported here, however, the agents’ positions are modeled continuously. In Jones’ work originating in image processing the agents are said to cover small squares (pixels) that can only hold one agent at a time. In this work, the agents have real valued positions  $\mathbf{x} \in \mathbb{R}^2$ . As I am simulating this model on a computer I end up with a discrete representation making my approach similar to that of Jones. However, the resolution of positions is much higher here and the observed patterns are different. My model is microscopic (agent positions, directions, and interactions are explicitly modeled). Microscopic modeling of self-organizing swarms goes at least back to [10]. The field of physics called *self-driven particle systems*, which was started by [13], is related (see also [1]).

The patterns shown by my model are similar to those found in stone and soil formations and were investigated by [6]. This similarity indicates that my model shows natural and possibly generalizable behaviors despite its abstractness.

The investigated system shows a wide variety of patterns. However, it turns out that most of the irregular patterns are transient. The patterns found in the transient are more complex than those the system converges to. Many dynamical systems show transient behavior in the beginning before they converge to a fixed point, periodic, or chaotic behavior (e.g., differential equations [7], coupled map lattices [5], cellular automata [14], or simple mappings [8]). Traditionally, transient phenomena are not investigated in fields such as mathematics (e.g., non-linear dynamics) or physics because they are a temporary phenomenon before a steady state is reached. Therefore, they were not considered to be interesting for research. However, it turned out that many important phenomena are transient but should be investigated as explained by [9]:

After I had presented my own lecture on irreversible thermodynamics, the greatest expert in the field of thermodynamics made the following comment: ‘I am astonished that this young man is so interested in nonequilibrium physics. Irreversible processes are transient. Why not wait and study equilibrium as everyone else does?’ I was so amazed at this response that I did not have the presence of mind to answer: ‘But we are all transient. Is it not natural to be interested in our common human condition?’

I believe the transient behavior observed in the model presented in this paper is relevant because many complex self-organized phenomena seem to be based on transient behavior [3, 2, 11, 9].

As we are more interested in complex patterns rather than regular or trivial patterns the question arises whether there exist parameter settings such that permanent complex patterns develop. Or will all systems always degenerate to trivial patterns after a transient? If self-organized complex patterns were always

transient we would like to control the transient length. This would be a critical issue for all evolved and engineered swarm-like systems.

## 2 The Multi-Agent System

In the following, the investigated multi-agent or swarm system is defined. The agents move in two-dimensional space. The change of a single agent's position  $\mathbf{x}$  over time is defined by

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} v, \quad (1)$$

for constant velocity  $v$ . The change of the agent's direction  $\phi$  over time is defined by

$$\frac{d\phi}{dt} = \alpha(s_l(t), s_c(t), s_r(t)) \Delta(t), \quad (2)$$

for  $\alpha(s_l(t), s_c(t), s_r(t)) \in \{-1, 0, 1\}$  defining the direction of the turns (clockwise or counterclockwise),  $\Delta$  defining the absolute value of the turn angles, and sensor values  $s_c$ ,  $s_l$ , and  $s_r$  defined by

$$s_c(t) = P \begin{pmatrix} x_1 + \cos(\phi)d \\ x_2 + \sin(\phi)d \end{pmatrix}, \quad (3)$$

$$s_l(t) = P \begin{pmatrix} x_1 + \cos(\phi - \sigma)d \\ x_2 + \sin(\phi - \sigma)d \end{pmatrix}, \quad (4)$$

$$s_r(t) = P \begin{pmatrix} x_1 + \cos(\phi + \sigma)d \\ x_2 + \sin(\phi + \sigma)d \end{pmatrix}, \quad (5)$$

for a potential field  $P$  representing the environment,  $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ , sensor angle  $\sigma$ , and sensor distance  $d$ . Closely following [4], we define

$$\alpha(s_l(t), s_c(t), s_r(t)) = \begin{cases} 0, & \text{for } s_c(t) > s_l(t) \wedge s_c(t) > s_r(t) \text{ (no turn)} \\ \pm 1, & \text{for } s_c(t) < s_l(t) \wedge s_c(t) < s_r(t) \text{ (random turn)} \\ +1, & \text{for } s_l(t) < s_r(t) \text{ (right turn)} \\ -1, & \text{for } s_r(t) < s_l(t) \text{ (left turn)} \end{cases}. \quad (6)$$

The random turn has a probability of 50% for  $+1$  and 50% for  $-1$ . We define

$$\Delta(t) = \begin{cases} \phi_{\text{rot}}, & \text{for } t \in \{0, \tau, 2\tau, \dots\} \\ 0, & \text{else} \end{cases}, \quad (7)$$

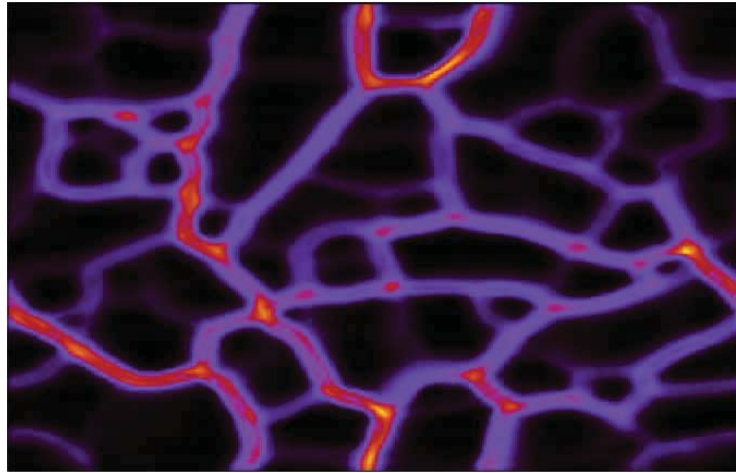
for a constant rotation angle  $\phi_{\text{rot}}$  and a time interval  $\tau$  at which the agents turn and their directions are updated (in this work we set  $\tau = 1$ ). Thus, we obtain a synchronized system that is discrete in time. The system could, for example, be extended by defining  $\Delta$  as a stochastic process. This would transform eq. 2 in a stochastic differential equation.

The temporal evolution of the potential field  $P$  is, in principle, defined by the following partial differential equation

$$\frac{\partial P(\mathbf{x}, t)}{\partial t} = D \nabla^2 P(\mathbf{x}, t) - \eta P(\mathbf{x}, t) + \theta \sum_{i=1}^N \delta(\mathbf{x} - \mathbf{x}_i(t)), \quad (8)$$

**Table 1:** Typical parameters.

sensor angle $\sigma$	$45^\circ$
rotation angle $\phi_{\text{rot}}$	$45^\circ$
rotation period $\tau$	1 [time units]
grid length $L$	400
sensor distance $d$	0.0375 [units]
velocity $v$	0.01 [units/ $\tau$ ]
diffusion $D$	0.1 [(1/ $L$ )/(10 $\tau$ )]
evaporation $\eta$	0.04 [1/(10 $\tau$ )]
addition $\theta$	5 [1/ $\tau$ ]
number of agents $N$	1000
active cell threshold $\delta_{\text{active}}$	$N/100$
simulated steps	$1 \times 10^5$ [time units]


**Figure 1:** Typical example of a complex, most likely transient pattern for  $N = 6750$ ,  $L = 600$ ,  $t = 3000$ .

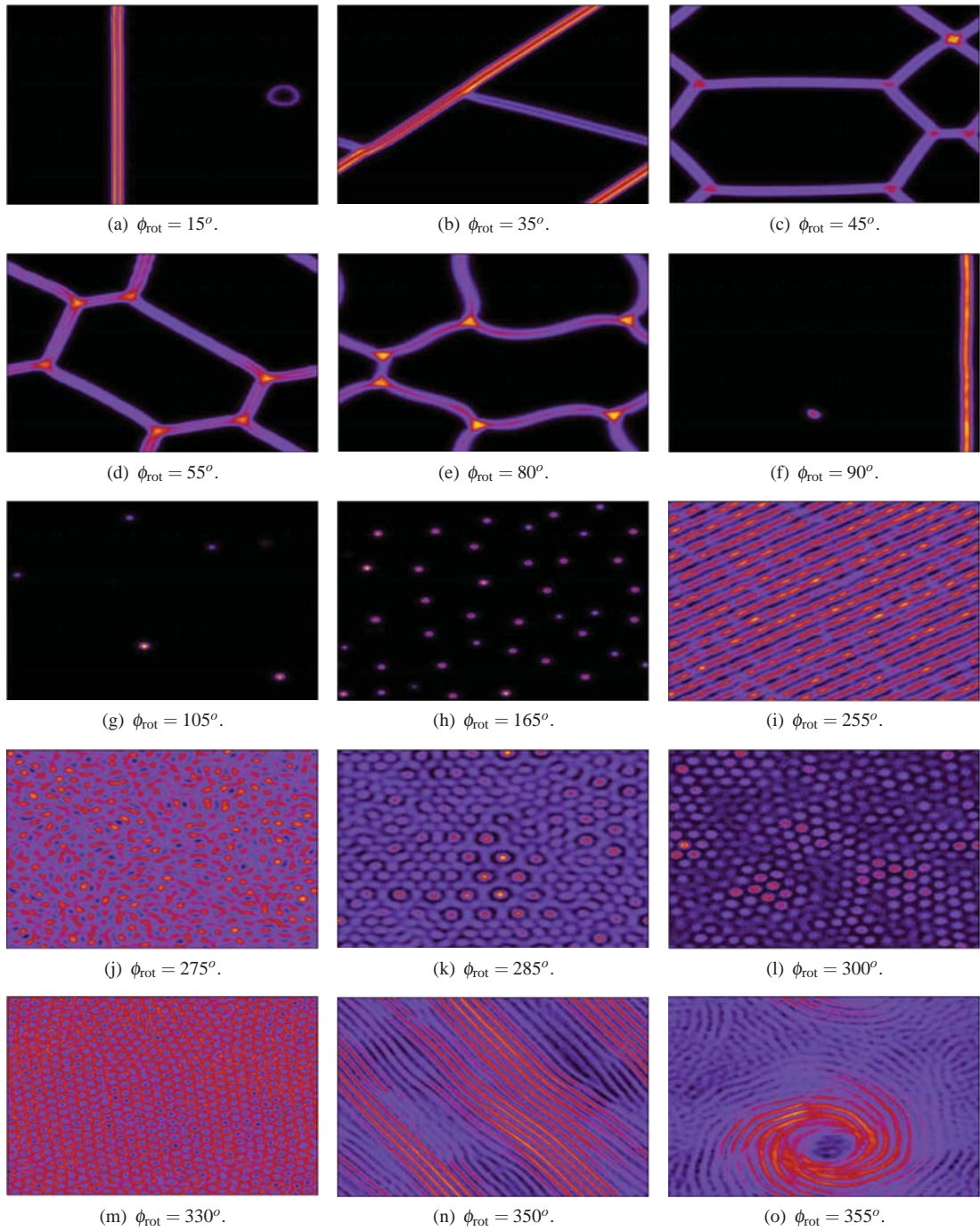
for diffusion  $D$ , evaporation rate  $\eta$ , addition  $\theta$  (the Dirac delta indicates that an agent only contributes to the potential field at its position), number of agents  $N$ , and agent positions  $\mathbf{x}_i(t)$ . However, for simplicity and to reduce the computational complexity the diffusion and the evaporation (first two terms in eq. 8) are only executed at  $t \in \{0, 10\tau, 20\tau, \dots\}$  unlike the addition process (third term in eq. 8) that is executed at  $t \in \{0, \tau, 2\tau, \dots\}$ . Furthermore, in my implementation the diffusion is not normalized in correspondence to the resolution of the used grid that is used as spatial discretization. The grid is always quadratic and the number of grid points is given by  $L^2$ . Periodic boundary conditions apply (i.e., space is a torus).

The observed behavior is based on the nonlinearity of the turns (jumps in the direction). The system is defined to be deterministic in most cases except for symmetry breaking situations (random turns defined by eq. 6). Thus, the system's dynamics is mostly determined by the inhomogeneities in the initialization.

### 3 Pattern Formation

Table 1 gives the typical parameters that were used, if not explicitly stated otherwise. In Fig. 1 a typical example of a complex, most likely transient pattern is shown ( $N = 6750$ ,  $L = 600$ ,  $t = 3000$ ). Note that the agents tend to move in parallel nearby lines. This seems to be a qualitative difference to the patterns reported by [4] and might, therefore, be induced by the higher resolution of agent positions (continuous positions approximated by floating point arithmetic).

Fig. 2 shows typical examples of the patterns in the potential field obtained by different rotation angles  $\phi_{\text{rot}}$ . These plots demonstrate the variety of patterns that the investigated systems produces.



**Figure 2:** Patterns for varied rotation angle  $\phi_{\text{rot}}$ .

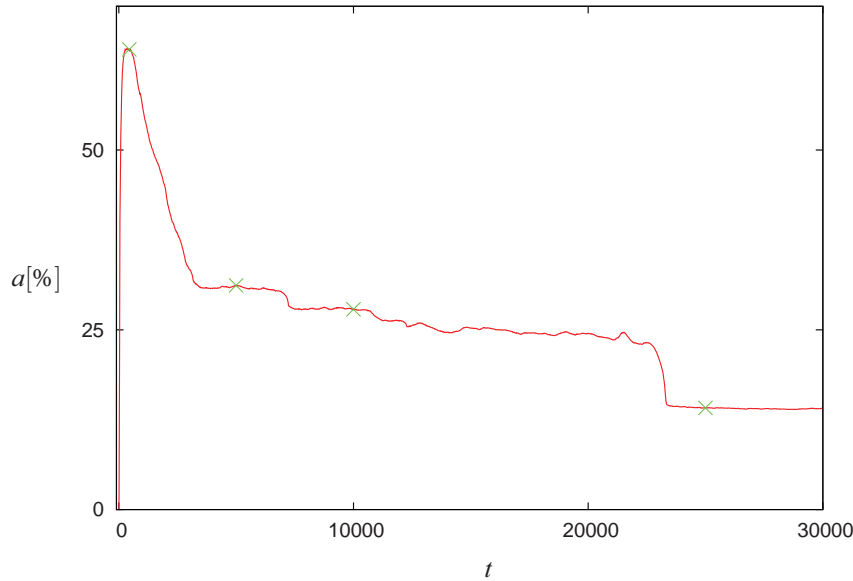


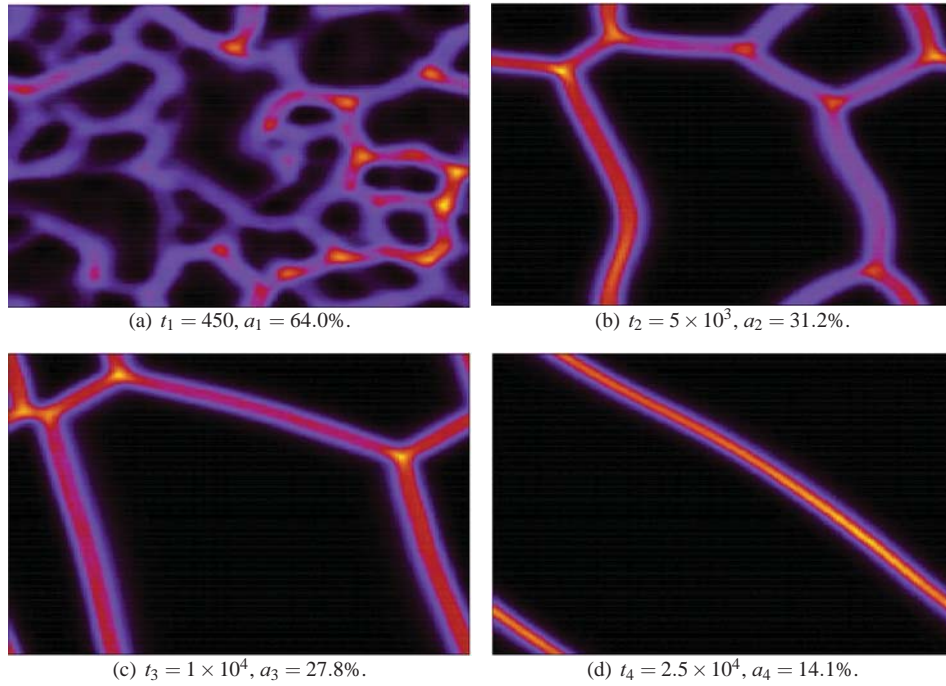
Figure 3: Temporal evolution of the active cell percentage  $a$  for  $L = 150$ .

## 4 Investigation of the Transients

In the following we want to investigate the transient behavior of this system. We need a measure of complexity to analyze the dynamics of the transient. A subjective measurement of the pattern complexity is the simple estimation by looking at the plots of the potential field. However, we need an objective and automatable metric to measure the pattern complexity. We use a simple metric that just computes the percentage  $a$  of grid points of the potential field that are above a threshold  $\delta_{\text{active}}$  (called *active cells* in the following). We obtained good results by setting  $\delta_{\text{active}} = N/100$ . This simple metric is a valid measure of complexity for the investigated parameter sets because complex patterns are composed of many cells with values above the threshold, while simple patterns have fewer active cells. In general a high active cell percentage does not assure a complex pattern. However, in the parameter region, that is investigated in the following, this is assured by simulation. Moreover, small changes in the temporal evolution of the active cell percentage correlate with a static pattern in the potential  $P$ . Thus, the active cell percentage indicates quasi-stationary and stationary states.

A typical example for  $L = 150$  is shown in Fig. 3. The percentage of active cells decreases over time until a steady state is reached. Fig. 4 shows four plots of the potential field of the four times marked in Fig. 3 ( $t_1 = 450$ ,  $t_2 = 5 \times 10^3$ ,  $t_3 = 1 \times 10^4$ , and  $t_4 = 2.5 \times 10^4$ ).

It is well known that the transient times scale exponentially with the dimensionality in many high dimensional systems (e.g., see [12]). In the following, we want to estimate a lower bound of the average transient time depending on the system's size which is the grid length  $L$  here. We do this by measuring the time until a pattern without bifurcations is reached (c.f. Fig. 4(d)) for several samples. We believe that such bifurcation-free patterns are a steady state for systems of grid size  $L \leq 200$  which is supported by my experience with long runs of the simulator. However, it is in principal possible that fluctuations lead to the formation of a new (and possibly stable) bifurcation albeit I have never observed that. In order to get an objective measure of when exactly a bifurcation-free pattern is reached and the simulation can be stopped, I use the active cell percentage  $a$  again. For each grid length  $L$  a threshold  $a_{\text{thresh}}^L$  is determined that characterizes the upper bound of the active cell percentage for bifurcation-free patterns. The bifurcation-free pattern of highest active cell percentage is a long line that twines diagonally around the torus. Also several unconnected lines occur (usually not more than two for the investigated grid lengths) that also have a high active cell percentage. For the five investigated grid lengths ( $L \in \{100, 125, 150, 175, 200\}$ ) we got thresholds that decrease approximately proportional to increased grid lengths ( $a_{\text{thresh}} \in \{31\%, 25\%, 20\%, 18.5\%, 17\%\}$ ). Furthermore, we used an additional parameter to increase the detection rate of bifurcation-free patterns.



**Figure 4:** Four plots of the potential field of the example shown in Fig. 3.

We determine the order in the agents' directions by determining the length of the sum of the direction vectors of all agents:

$$\left\| \sum_i \begin{pmatrix} \cos \phi_i \\ \sin \phi_i \end{pmatrix} \right\|. \quad (9)$$

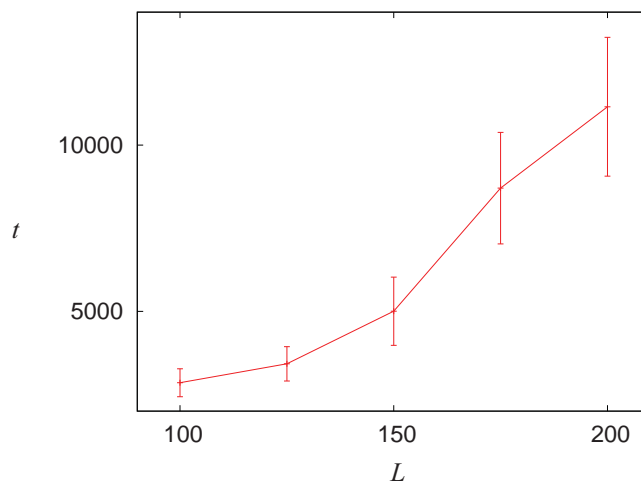
If this value got bigger than 0.4 (i.e., a high percentage of the swarm moves in similar directions) I had the simulation stopped as well. For grid lengths of  $L \geq 150$  we found patterns with bifurcations for which we could not determine within  $t \leq 1 \times 10^6$  whether they are unstable and degenerate to a bifurcation-free pattern. These samples were thrown out and were not included in the calculation of the lower bound of the average transient time. The result is shown in Fig. 5 and it suggests an exponential increase of the actual average transient time.

In contrast to Fig. 3, the behavior of the system is more complex for bigger grid sizes and bigger agent numbers. In Fig. 6 we give two examples of the active cell percentage for a bigger system. In case of Fig. 6(a), the behavior is dynamic most of the time. New patterns emerge and break down again. Even values of  $a < 2\%$  are reached corresponding to patterns of very low complexity. However, these states are not stable and more complex patterns emerge again. Also time intervals of little dynamics are found (e.g.,  $t \approx 2 \times 10^5$ ,  $t \approx 3 \times 10^6$ ,  $t \approx 5.2 \times 10^6$ ). Whether this irregular behavior is transient stays unanswered. In contrast, Fig. 6(b) gives an example of a system that most likely converges to a stable pattern after a long transient of about  $1.4 \times 10^6$  time steps.

## 5 Discussion

The vast variety of patterns found in this system is higher than expected because only one parameter was changed. Even small changes of the rotation angle lead to qualitative changes in the observed patterns in certain parameter intervals. This is also why we cannot tell whether all main classes of patterns are shown in this paper. A parameter interval, within which a potentially yet unreported pattern might be found, could be very small.

The degeneration of the patterns from complex to trivial over time corresponds to the second law of thermodynamics and to what we observe in our complex real world. This might be an over-interpretation of



**Figure 5:** Increase of a lower bound of the average transient time with increasing grid length  $L$  (i.e., increasing dimensionality of the systems). The error bars give the 95% confidence interval.

this model but it could be based on similar principles as the phenomena we observe in nature. At the current state of research, it stays unanswered whether there exist parameter settings that lead to permanent complex patterns. Due to the discrete simulation all patterns will at least become cyclic eventually, because the number of possible states is finite. However, acyclic patterns might exist in principle.

## 6 Summary and Outlook

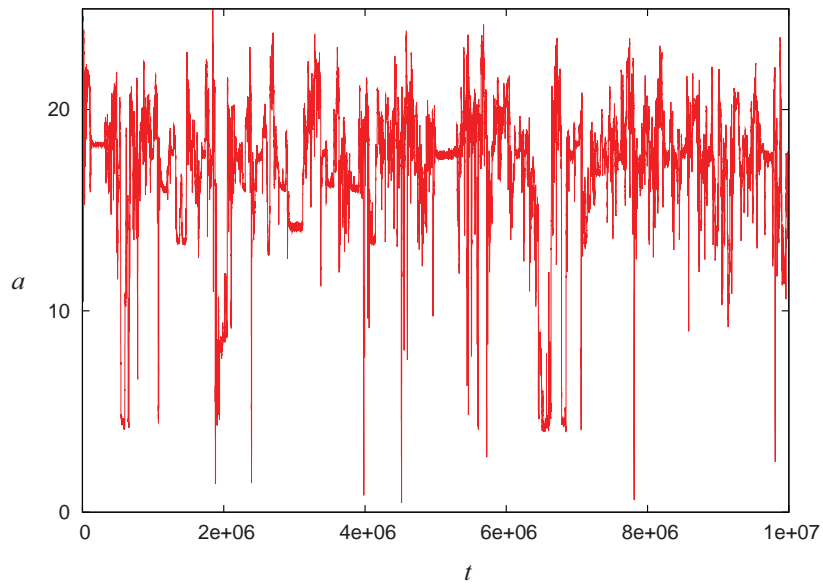
In this paper, a self-organized system is reported that generates a variety of complex patterns. At least for small system sizes these patterns are transient. A lower bound of the transient time was shown that suggests an exponential increase in the average transient time.

The presented model seems to be a good object of research to investigate the relevance and influence of transient behavior in self-organized swarms.

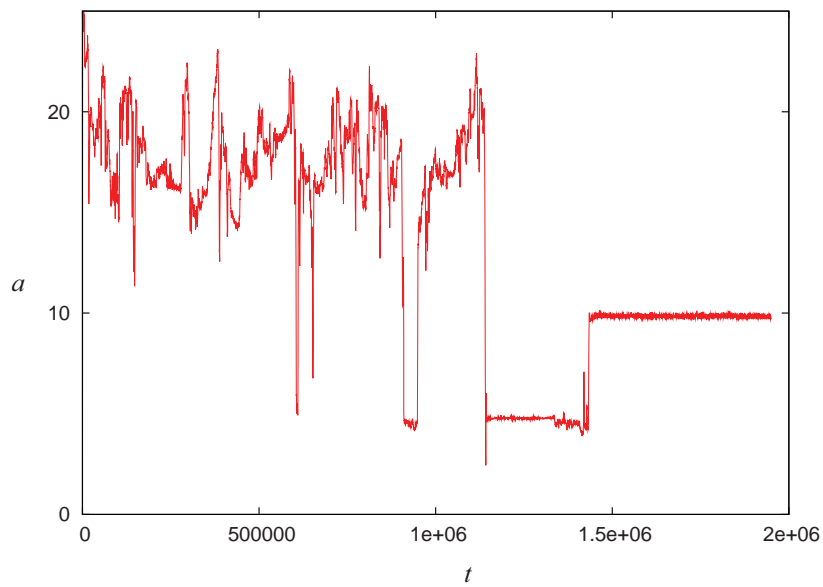
An investigation of the influence of noise to this system is pending. In first experiments, no qualitative changes for reasonable intensities of noise were found concerning the formation of patterns and the influence to transient lengths. This will be the focus of future work.

## 7 References

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(a)



(b)

**Figure 6:** Examples of the temporal evolution of the active cell percentage  $a$  for grid length  $L = 400$  and number of agents  $N = 3000$ .



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