I. Troch, F. Breitenecker, eds. ISBN 978-3-901608-35-3

STABILITY ANALYSIS OF PIEZOELASTIC STRUCTURES FOR ROBUST VIBRATION CONTROL

T. Rittenschober
Profactor Production Research GmbH, Austria

Corresponding author: T. Rittenschober,
Profactor Production Research GmbH, Smart and Adaptive Structures Group,
4407 Steyr/Gleink, Im Stadtgut A2, Austria, thomas.rittenschober@profactor.at

Abstract. The design of an observer for self sensing actuation of functional materials requires acurate modeling of the piezoelastic structure under investigation. Under the assumption of linear piezoelasticity and Kirchhoff plate theory, the resulting mathematical structure facilitates controller design for harmonic disturbance suppression. Stability issues of the infinite dimensional model are resolved in a straightforward manner by application of results on positive real transfer functions in conjunction with the small gain theorem. If the disturbance frequency is unknown, the proposed controller can be extended by an ordinary differential equation for the unknown frequency which is derived using a stability argument involving the Hamilton-Jacobi inequality.

1 Introduction

Compensating for harmonic vibrations induced by rotating machinery and other periodic disturbances is a common problem in the industry and robust controller solutions accompanied by self sensing actuation for lightweight mechanical constructions, are essential to the breakthrough of active technologies in industrial applications. In fact, self sensing actuation of functional materials enables collocation of actuator and sensor. In the case of piezoelectric actuators, self sensing requires a robust separation of the time derivative of the actuation voltage from the measured electric current in order to obtain a signal which is proportional to the strain rate integrated over the piezoelectric patch area. Due to the unfavorable ratio of these two signals, the design of an observer for the electric current due to the direct piezoelectric effect seems most appropriate. The design of observers relies on accurate modelling of the underlying piezoelastic structure which in our case is chosen to be a rectangular plate equipped with two piezoelectric actuators and two opposite edges either clamped or free. When designing a controller for the purpose of suppressing harmonic vibrations, i.e. we require the regulated output to tend to zero as time goes to infinity, the special mathematical structure of the model tremendously facilitates controller synthesis and resolves stability issues of the infinite-dimensional model in a straightforward manner by applying results on positive real transfer functions in conjunction with the small gain theorem. This result has been reported on a previous occasion, see [7]. In many cases, however, the disturbance frequency is unknown. In this case, the proposed controller can be extended by a nonlinear ordinary differential equation for the unknown frequency where the proof of stability of the infinite-dimensional closed loop involves the Hamilton Jacobi inequality together with the small gain theorem.

2 The Mathematical Model

The plate structure under investigation along with its geometric properties is depicted in figure 1. Geometric parameters and material properties are given by $a = 0.45m$, $b = 0.35m$, $c = 0.1m$, $d = 0.1525m$, $e = 0.045m$, $f = 0.01m$, h_b (thickness of plate) = $0.001m$, h_p (thickness of patch) = $0.001m$, $k = 0.05m$, $l = 0.55m$, ρ (mass density of plate) = $7500kg/m^3$, E_b (Young's modulus plate)= $2e11N/m^2$, E_p (Young's modulus patch)= $6e10N/m^2$, ν_b (Poisson's ratio of plate) = 0.33, ν_p (Poisson's ratio of patch) = 0.25, G^{113} (piezoel. coupling const.) = $6.62C/m^2$ and F^{33} (relative permittivity) = 1.21e-8F/m.

Under the assumption of linearized piezoelasticity and the strain displacement relations as introduced by Kirchhoff, the piezoelastic plate-like structure under consideration may be modelled according to the partial differential equation, see [2],

$$\mu \frac{\partial^2}{\partial t^2} w + \bar{D} \left(\frac{\partial^4}{\partial X^4} + 2 \frac{\partial^4}{\partial X^2 \partial Y^2} + \frac{\partial^4}{\partial Y^4} \right)(w) = cU \left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right)(\lambda) - Bd \tag{1}$$

where the coordinate t represents time, X, Y, Z are local coordinates for the body $\mathscr{B} \subseteq \mathbb{R}^3$, $w = w(t, X, Y)$ is the transverse displacement of the plate's neutral fibre, $\mu = \rho h_b$ is the mass area density of the respective material, $\bar{D}$ is the flexural rigidity of the plate and computes as $\bar{D} = E_b h_b^3 / \left(12\left(1 - \nu_b^2\right)\right)$. We denote by E_b and h_b Young's modulus and the thickness of the support structure, respectively, and ν_b is Poisson's ratio. The associated material and geometric properties of the piezoelectric layer are denoted by a subscript p. We shall note at this stage that the structural influence due to added mass and stiffness from the piezoelectric material will be neglected. Assuming a typical property of transversely isotropic piezoelectric material that the coupling constants G^{113} and G^{223} appearing

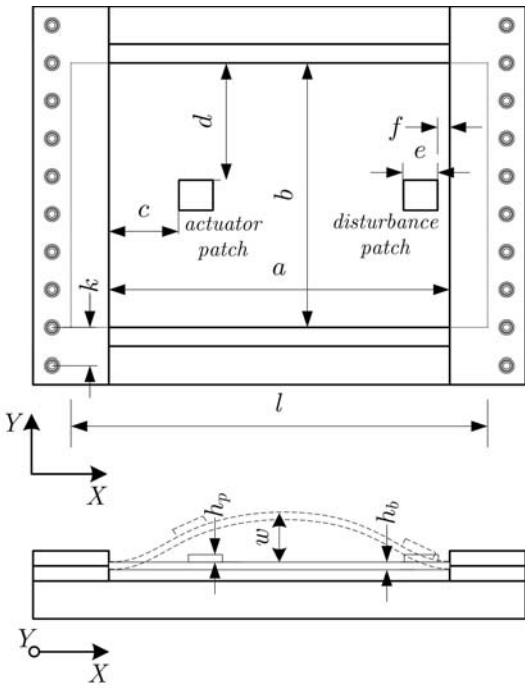

Figure 1: Geometric configuration of the piezoelastic structure.

in the constitutive equations [6]

$$\sigma = C\varepsilon - GE\ ,\quad D = G\varepsilon + FE \tag{2}$$

are identical, the constant c then computes as $c = G^{113}h_p\left(h_p + 2h\right)/4$ and, hence, incorporates piezoelectric and geometric properties. The constant h describes the distance between the neutral fibre and the interface between the piezoelectric layer and the support structure. We denote by $\sigma,\ \varepsilon,\ C,\ F,\ G,\ E$ and D in (2) the stress, strain, elasticity, relative permittivity, coupling, electric field and electric flux density tensors, respectively. $U = U(t)$ is the electric potential at the upper electrode of the piezoelectric layer, the shape function $\lambda = \lambda(X,Y)$ describes the weighted spatial distribution of the electric potential U at the upper electrode and $B = B(X,Y)$ is the spatial distribution of the body force acting on the structure. The tensorial components appearing in the constitutive equations (2) are, in general, functions of the spatial coordinates and will be set piecewise constant in our case.
The solution technique applied to the partial differential equation from (1) under the appropriate kinematic and dynamic boundary and initial conditions makes use of separation of variables, i.e.

$$w(t,X,Y) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} q_{ij}(t)\,\phi_{ij}(X,Y)\ ,$$

the orthogonality property of the eigenfunctions ϕ_{ij} and the Laplace transform, see [1]. The transfer functions of the system $G_a\left(s,X^1,X^2\right)$ due to piezoelectric actuation may be then written as

$$G_a(s,X,Y) = \frac{\hat{w}(s,X,Y)}{\hat{U}(s)} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\phi_{ij}P_{a,ij}\frac{1}{\frac{s^2}{\omega_{ij}^2} + 2\xi_{ij}\frac{s}{\omega_{ij}} + 1} \tag{3}$$

with

$$P_{a,ij} = \frac{c}{\omega_{ij}^2}\int_{\mathscr{D}}\phi_{ij}\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)(\lambda)\,\Omega$$

where $\mathscr{D}$ denotes the corresponding domain of integration, $\Omega = \mathrm{d}X\mathrm{d}Y$ is the corresponding volume form and $P_{a,ij}$ are the modal gains of mode ij of the transfer function due to piezoelectric actuation. We use $\phi_{ij}(X,Y) = \phi_i^1(X)\,\phi_j^2(Y)$ where $\phi_i^1(X)$ and $\phi_j^2(Y)$ are chosen as the fundamental mode shapes of beams having the boundary conditions of the plate [5]. The natural frequencies ω_{ij} for the C-F-C-F configuration are computed according to [5] and ξ_{ij} is the damping ratio associated with mode mode ϕ_{ij}. The type of damping involved is referred to as Rayleigh damping which is assumed to be proportional to the distributed mass and stiffness of the structure. Due to the non-continuity and, hence, non-differentiability of the chosen shape function λ at the edges of the patches, we have to take a closer look at the modal gains $P_{a,ij}$. The relation

$$\int_{\mathscr{D}}\phi_{ij}\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)(\lambda)\,\Omega = \int_{\mathscr{D}}\lambda\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)(\phi_{ij})\,\Omega$$

is derived from integration by parts and only holds if the corresponding boundary integral vanishes. Alternatively, one may divide the domain of the plate into subdomains of constant electric potential and connect the subdomains via the governing kinematic and dynamic boundary conditions. In this case, the governing pde (1) of the subdomain becomes homogeneous with respect to piezoelectric actuation and the piezoelectric material only acts as a distributed torque at the boundary.

3 The Self Sensing Concept

Considering the linear constitutive relations from (2), respecting conservation of charge, i.e.

$$\frac{\partial}{\partial X}D^1+\frac{\partial}{\partial Y}D^2+\frac{\partial}{\partial Z}D^3=0$$

which reduces to $\frac{\partial}{\partial Z}D^3=0$ due to the simplification $\frac{\partial}{\partial X}\Phi=\frac{\partial}{\partial Y}\Phi=0$ on the electrical potential Φ and $G^{111}=G^{112}=0$ of typical piezoelectric material, by Kirchhoff's assumptions on the relations of strain and displacement and $G^{123}=G^{213}=0$, we get

$$D^3=-G^{113}Z\frac{\partial^2}{\partial X^2}w-G^{223}Z\frac{\partial^2}{\partial Y^2}w+F^{33}E_3\ .$$

Accounting for $G^{113}=G^{223}$, the charge Q at the electrode of the piezoelectric actuator computes as

$$Q=\int_{\mathscr{D}}D^3\Omega=\frac{F^{33}e^2}{h_p}U-\frac{1}{2}G^{113}\left(2h+h_p\right)\int_{\mathscr{D}}\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}\right)(w)\,\Omega\ .$$

The constant $C_p=F^{33}e^2/h_p$ is the capacitance of the piezoelectric patch.

Alternatively, if the electric current $I=I_{indirect}+I_{direct}=\frac{\mathrm{d}}{\mathrm{d}t}Q$, i.e.

$$I=\frac{F^{33}e^2}{h_p}\frac{\mathrm{d}}{\mathrm{d}t}U-\frac{1}{2}G^{113}\left(2h+h_p\right)\int_{\mathscr{D}}\left(\frac{\partial^3}{\partial t\partial X^2}+\frac{\partial^3}{\partial t\partial Y^2}\right)(w)\,\Omega$$

is at our disposal, we only need to robustly separate the time derivative of actuation voltage from it in order to obtain a signal which is proportional to the strain rate integrated over the patch area. Due to the unfavorable ratio of these two signals, the design of an observer for the direct part of the electric current seems most appropriate. The corresponding transfer function of such a model is given by

$$\bar{G}_a(s)=\frac{\hat{I}(s)}{\hat{U}(s)}=\frac{C_ps}{\frac{s^2}{\omega_C^2}+2\xi_C\frac{s}{\omega_C}+1}+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{2h_pP_{a,ij}^2s}{\frac{s^2}{\omega_{ij}^2}+2\xi_{ij}\frac{s}{\omega_{ij}}+1}\ . \tag{4}$$

The state space representation is written as

$$\begin{aligned}
\tfrac{\mathrm{d}}{\mathrm{d}t}x_{m,ij} &= A_{m,ij}x_{m,ij}+b_mu\ ,\quad i,j=1,\ldots,\infty\\
\tfrac{\mathrm{d}}{\mathrm{d}t}x_C &= A_Cx_C+b_Cu\\
\\
y_{m,ij} &= c_{m,ij}^Tx_{m,ij}\ ,\quad i,j=1,\ldots,\infty\\
y_C &= c_C^Tx_C\\
y &= y_C+\textstyle\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}y_{m,ij}\\
e &= \textstyle\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}y_{m,ij}
\end{aligned} \tag{5}$$

where

$$A_{m,ij}=\begin{pmatrix}0 & 1\\ -\omega_{ij}^2 & -2\xi_{ij}\omega_{ij}\end{pmatrix},\ A_C=\begin{pmatrix}0 & 1\\ -\omega_C^2 & -2\xi_C\omega_C\end{pmatrix},$$

$$b_m=b_C=\begin{pmatrix}0\\1\end{pmatrix},\ c_{m,ij}=\begin{pmatrix}0\\ \bar{P}_{a,ij}\omega_{ij}^2\end{pmatrix},\ c_C=\begin{pmatrix}0\\ C_p\omega_C^2\end{pmatrix}$$

with $\bar{P}_{a,ij}=2h_pP_{a,ij}^2$. We denote by $u\in\mathbb{R}$ the driving voltage and by $y\in\mathbb{R}$ the measured current, i.e. the electric current I. The regulated output $e\in\mathbb{R}$ is the sum of the direct electric currents $y_{m,ij}\in\mathbb{R}$ due to each mode ij, $A_{m,ij}\in\mathbb{R}^{2\times2}$ and $b_m,c_{m,ij}\in\mathbb{R}^2$ model the direct piezoelectric effect with state vector $x_{m,ij}\in\mathbb{R}^2$. The objects $A_C\in\mathbb{R}^{2\times2}$ and $b_C,c_C\in\mathbb{R}^2$ are the dynamic matrix and the input describing the capacitive effect of the piezoelectric actuator with state vector $x_C\in\mathbb{R}^2$. The indirect electric current is denoted by $y_C\in\mathbb{R}$. The model (4) can also be descibed by its PCHD representation [8].

In equation (4), we have extended the dynamics due to the indirect piezoelectric effect by taking resistance R_p and inductance L_p of the piezoelectric patch into account, where

$$\omega_C = \frac{1}{\sqrt{L_p C_p}}\ ,\ \xi_C = \frac{1}{2} R_p \sqrt{\frac{C_p}{L_p}}.$$

The corresponding equivalent circuit diagram for (4) is depicted in figure 2 describing the indirect and direct piezoelectric effect by means of a parallel conncetion of an RLC oscillator and an infinite series of damped spring mass oscillators, respectively.

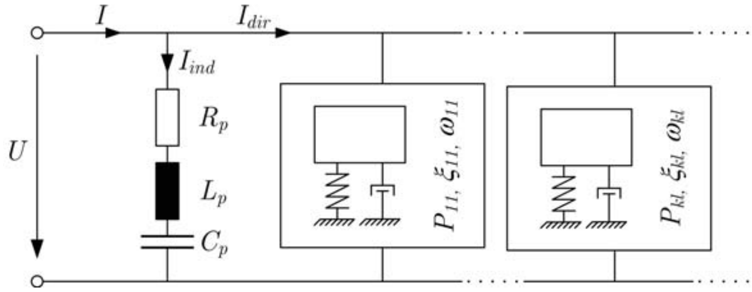

Figure 2: Equivalent circuit diaram for transfer function (4) describing the additive nature of the indirect and direct piezoelectric effect.

4 Stability in Robust Vibration Control

A preliminary step in designing a feedback controller for vibration suppression requires the estimation of the current due to the direct piezoelectric effect. Since the system (4) is infinite dimensional, one cannot construct a realizable observer for (4), but one can construct a finite dimensional approximation [8]. The state space representation of the Kalman filter is given by

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\hat{x}_{m,ij} &= A_{m,ij}\hat{x}_{m,ij} - l_{m,ij}\sum_{I=1}^{k}\sum_{J=1}^{l} c_{m,IJ}^T \hat{x}_{m,IJ} - l_{m,ij} c_C^T \hat{x}_C + b_m u + l_{m,ij} y \\
\frac{\mathrm{d}}{\mathrm{d}t}\hat{x}_C &= \left(A_C - l_C c_C^T\right)\hat{x}_C - l_C \sum_{I=1}^{k}\sum_{J=1}^{l} c_{m,IJ}^T \hat{x}_{m,IJ} + b_C u + l_C y \\
\hat{e} &= \sum_{i=1}^{k}\sum_{j=1}^{l} c_{m,ij}^T \hat{x}_{m,ij}
\end{aligned}$$

where $L = \left(l_{m,11}^T, \ldots, l_{m,kl}^T, l_C^T\right)^T$ with $l_{m,kl}, l_C \in \mathbb{R}^2$ denotes the observer gain, $\hat{x}_{m,ij}, \hat{x}_C \in \mathbb{R}^2$, $i = 1, \ldots, k$ and $j = 1, \ldots, l$.

Closing the loop is achieved by a controller for the robust suppression of harmonic vibrations where the underlying theory is described in [4] and [8].

The proposed controller for the robust suppression of harmonic vibrations exhibits the structure

$$\begin{aligned}
\tfrac{\mathrm{d}}{\mathrm{d}t}\xi_1 &= \Phi\xi_1 + \Theta\hat{e} \\
u_1 &= \Gamma\xi_1 + u_2
\end{aligned} \tag{6}$$

and the memoryless stabilizer

$$u_2 = K_2 \hat{e}\ ,\ \ K_2 > 0. \tag{7}$$

with the state $\xi_1 \in \mathbb{R}^2$, $\Phi \in \mathbb{R}^{2\times 2}$, $\Theta_1 \in \mathbb{R}^{2\times 1}$, $\Gamma \in \mathbb{R}^{1\times 2}$, the outputs $u_1, u_2 \in \mathbb{R}$ and

$$\Phi = \begin{pmatrix} 0 & 1 \\ -\omega_d^2 & 0 \end{pmatrix},\ \Theta_1 = \begin{pmatrix} 0 \\ K \end{pmatrix},\ \Gamma = \begin{pmatrix} 0 & 1 \end{pmatrix}, \tag{8}$$

The controller is driven by the observed direct part of the electric current $\hat{e}$.

The proof of stability for the infinite dimensional piezoelastic structure (4) together with the feedback configuration consisting of the Kalman filter for the finite dimensional part of model (4), $G_{a,fin}$,

$$\bar{G}_{a,fin}(s) \quad = \quad \frac{\hat{I}_{fin}(s)}{\hat{U}(s)} = \bar{G}_{a,ind} + \bar{G}_{a,dir}$$

$$= \frac{C_p s}{\frac{s^2}{\omega_C^2} + 2\xi_C \frac{s}{\omega_C} + 1} + \sum_{i=1}^{k} \sum_{j=1}^{l} \frac{2h_p P_{a,ij}^2 s}{\frac{s^2}{\omega_{ij}^2} + 2\xi_{ij} \frac{s}{\omega_{ij}} + 1}.$$

and the controller (6) is described in [7]. In brief, the finite gain L_2 stability of the finite dimensional part of the closed loop is shown using a passivity based approach and the proof of I/O stability of the corresponding infinite-dimensional closed loop involves the small gain theorem with the infinite-dimensional part $\bar{G}_{a,\text{inf}}$ of (4)

$$\bar{G}_{a,\text{inf}}(s) = \frac{\hat{I}_{\text{inf}}(s)}{\hat{U}(s)} = \sum_{i=k+1}^{\infty} \sum_{j=l+1}^{\infty} \frac{2h_p P_{a,ij}^2 s}{\frac{s^2}{\omega_{ij}^2} + 2\xi_{ij} \frac{s}{\omega_{ij}} + 1}$$

which we pull out using the linear fractional transformation

$$\begin{pmatrix} \hat{U} \\ \hat{e} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} \hat{I}_{\text{inf}} \\ \hat{d} \end{pmatrix}$$

where d is the disturbance acting on the measured output I.

If the disturbance frequency is unknown, we may be well advised to extend the proposed controller by some sort of adaptive control. The governing nonlinear ordinary differential equation for the unknown frequency ω_d will be derived next using a stability argument. The full control concept is depicted in figure 3.

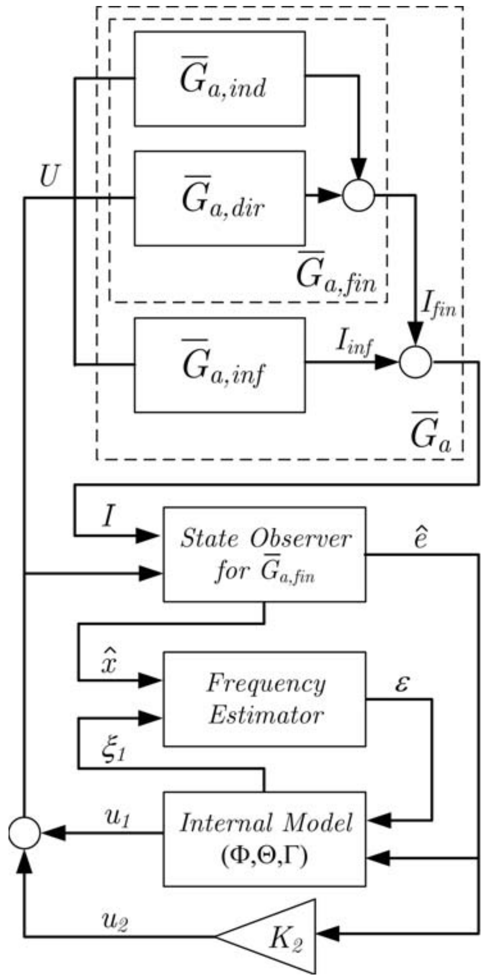

Figure 3: Control loop showing the functional blocks and signals involved in the robust suppression of (un)known harmonic disturbances.

We first observe that a necessary and sufficient condition for the L_2 gain of M_{11} to be less than or equal to γ_2 is the existence of a positive semidefinite solution P of the Riccati equation

$$PA_M + A_M^T P + \frac{1}{\gamma_2^2} P b_M b_M^T P + c_M c_M^T = 0 \tag{9}$$

which also satisfies the Hamilton-Jacobi equation

$$x^T P A_M x + \frac{1}{2\gamma_2^2} x^T P b_M b_M^T P x + \frac{1}{2} x^T c_M c_M^T x = 0\ , \tag{10}$$

see [3], where we have used

$$M_{11} = \left(\begin{array}{c|c} A_M & b_M \\ \hline c_M^T & 0 \end{array} \right)$$

with $A_M \in \mathbb{R}^{2(kl+2)+2 \times 2(kl+2)+2}$, $b_M \in \mathbb{R}^{2(kl+2)+2}$ and $c_M \in \mathbb{R}^{2(kl+2)+2}$. Let ε denote the difference between the squared nominal and estimated disturbance frequency, i.e. $\varepsilon = \hat{\omega}_d^2 - \omega_d^2$. Extending M_{11} by the frequency estimator, we get the affine input system

$$\begin{array}{ll} \frac{\mathrm{d}}{\mathrm{d}t}\bar{x} & = f(\bar{x}) + g\hat{I}_{\mathrm{inf}} \\ U & = h(\bar{x}) \end{array} \tag{11}$$

where $\bar{x} = (x, \varepsilon)$, $x = (x_{m,11}, \ldots, x_{m,kl}, x_C, \hat{x}_{m,11}, \ldots, \hat{x}_{m,kl}, \hat{x}_C, \xi_1)$ with $\bar{x} \in \mathbb{R}^{2(kl+2)+3}$ and

$$f(\bar{x}) = \left(\begin{array}{c} A_M x + \varepsilon B_\varepsilon x \\ k(\bar{x}) \end{array} \right), \ g = \left(\begin{array}{c} b_M \\ 0 \end{array} \right), \ h(\bar{x}) = c_M^T x \tag{12}$$

with $f, g, h \in \mathbb{R}^{2(kl+2)+3}$ and

$$B_\varepsilon = \left(\begin{array}{cc} 0 & 0 \\ 0 & \tilde{I} \end{array} \right), \ \tilde{I} = \left(\begin{array}{cc} 0 & 0 \\ -1 & 0 \end{array} \right).$$

Now, the extended system (11) is finite gain L_2 stable with L_2 gain less than or equal to γ_4 if there is a continuously differentiable, positive semidefinite function V that satisfies the Hamilton-Jacobi inequality

$$\frac{\partial V}{\partial \bar{x}} f + \frac{1}{2\gamma_4^2} \frac{\partial V}{\partial \bar{x}} g g^T \left(\frac{\partial V}{\partial \bar{x}} \right)^T + \frac{1}{2} h^T h \leq 0 \,, \tag{13}$$

see [3]. Let us choose V to be

$$V(\bar{x}) = \frac{1}{2} x^T P x + \frac{1}{2\gamma_3} \varepsilon^2 \tag{14}$$

with P being the solution of the Riccati equation (9) and $\gamma_3 > 0$. Plugging (12) and (14) into (13), we arrive at

$$x^T P A_M x + 1/(2\gamma_4^2) x^T P b_M b_M^T P x + \frac{1}{2} x^T c_M c_M^T x + \varepsilon \left(x^T P B_\varepsilon x + \gamma_3^{-1} k(\bar{x}) \right) \leq 0.$$

Setting $\gamma_4 = \gamma_2$ and considering the Hamilton-Jacobi equality (10), we get

$$\varepsilon \left(x^T P B_\varepsilon x + \gamma_3^{-1} k(\bar{x}) \right) \leq 0. \tag{15}$$

Since expression (15) is not necessarily sign definite, we choose

$$\frac{\mathrm{d}}{\mathrm{d}t} \varepsilon == k(\bar{x}) = -\gamma_3 x^T P B_\varepsilon x = -\gamma_3 x^T P \xi_1^1 \ , \ \ \gamma_3 > 0$$

to achieve equality in (13) and (15), respectively. Hence, system (11) is finite gain L_2 stable with L_2 gain γ_2. The stability of the infinite-dimensional closed loop can be proven by reinvoking the small gain theorem, i.e. the stability of the closed loop which now incorporates the nonlinear frequency estimator is guaranteed if and only if $\gamma_1 \gamma_2 < 1$ where γ_1, γ_2 are the H_∞ norms of $\bar{G}_{a,\mathrm{inf}}$ and the affine input system (11), respectively.

It is clear that we cannot arbitrarily choose the entries in P, since we have to respect the the positive semidefiniteness of P and we may only use the observer and regulator states $\left(\hat{x}_{m,11}, \ldots, \hat{x}_{m,kl}, \hat{x}_C, \xi_1 \right)$. The symmetric matrix P

$$P = \left(\begin{array}{cc} A & B \\ B^T & C \end{array} \right)$$

is positive semidefinite if and only if $A > 0$ and its Schur complement $C - B^T A^{-1} B$ in P is positive semidefinite. Every positive definite matrix A is invertible and its inverse A^{-1} is also positive definite, hence, $B^T A^{-1} B > 0$. Now, the choice

$$P = \left(\begin{array}{cc} P_{11} & P_{12} \\ P_{12}^T & p_{22} \end{array} \right)$$

with $P_{11} \in \mathbb{R}^{2(kl+2)+1 \times 2(kl+2)+1}$, $P_{12} \in \mathbb{R}^{2(kl+2)+1}$, $p_{22} \in \mathbb{R}$, $P_{11} = P_{11}^T > 0$, $P_{12}^T = [0, \ldots, 0, c_{m,11}^T, \ldots, c_{m,kl}^T, 0, \ldots, 0]$, $p_{22} \geq P_{12}^T P_{11}^{-1} P_{12}$ guarantees the positive semidefiniteness of P and immediately leads to the update law

$$\frac{\mathrm{d}}{\mathrm{d}t} \varepsilon = -\gamma_3 \xi_1^1 \left(\hat{e} + p_{22} \xi_1^2 \right) \ , \ \ \gamma_3 > 0 \tag{16}$$

which only uses the observed direct current $\hat{e}$ and the regulator state ξ_1 of (6).

5 Measurement Results

The actual implementation of the proposed observer and control algorithms are carried out on dSpace RTI 1104 rapid prototyping hardware. The power amplification for the piezoelectric patches is supplied by a Trek PZD 350 dual channel piezo driver which is equipped with a current monitor. The piezoelectric actuator at the boundary in-

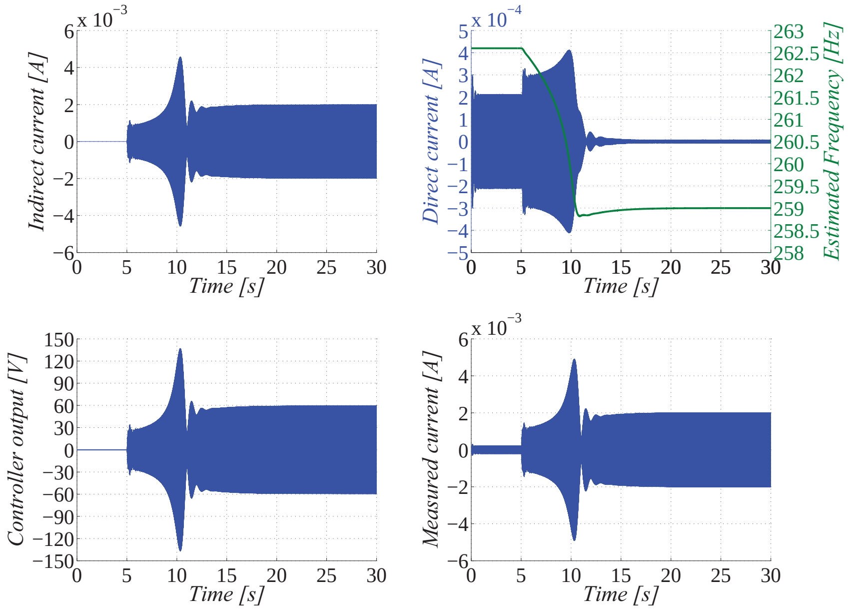

Figure 4: Time signals for direct part of the measured current, indirect part of measured current, controller output, actually measured current and estimated disturbance frequency when plate structure is excited at $\omega_d = 2\pi 259\ rad/s$ with frequency estimator initialized at $\omega_d(0) = 2\pi 262.6\ rad/s$.

jects a harmonic disturbance at $\omega_d = 2\pi 259\ rad/s$.For the case of an unknown harmonic disturbance, the controller (6) is extended by a frequency estimator. The corresponding sensor and control signals are shown and in figure 4 with control switched on after five seconds. The update law is initialized with a value of $\omega_d(0) = 2\pi 262.6\ rad/s$ and the estimated frequency progressively converges to the actual disturbance frequency of $2\pi 259\ rad/s$.

6 Summary

This contribution was concerned with compensation of harmonic vibrations in mechanical structures using the self sensing capability of piezoelectric actuators. In order to use self sensing for vibration control, an observer for the electric current due to the direct piezoelectric effect has been proposed. The special mathematical structure of the underlying piezoelastic structure facilitates controller synthesis for harmonic disturbance suppression and resolves stability issues of the infinite dimensional model by applying results on positive real transfer functions in conjunction with the small gain theorem. If the disturbance frequency is unknown, the proposed controller can be extended by an update law for the unknown frequency which has been derived using a stability argument involving the Hamilton-Jacobi inequality.

7 References

[1] S.J. Farlow. *Partial Differential Equations for Scientists and Engineers*. Wiley, New York, 1982.

[2] C.R. Fuller, S.J. Elliott SJ, P.A. Nelson. *Active Control of Vibration*. Academic Press, London, 1993.

[3] H.K. Khalil. *Nonlinear Systems*. Prentice Hall Inc., New Jersey, 1996.

[4] A. Isidori, L. Marconi L, A. Serrani. *Robust Autonomous Guidance - an Internal Model Approach*. Springer, London, 2003.

[5] A.W. Leissa. *Vibration of Plates*. Scientific and Technical Information Division, National Aeronautics and Space Administration, Washington D.C., 1969.

[6] W. Nowacki. *Dynamic Problems of Thermoelasticity*. Noordhoff International Publishing, Amsterdam, 1975.

[7] T. Rittenschober, K. Schlacher. Self sensing based active control of piezoelastic structures. 4^{th} *European Conference on Structural Control*, St. Petersburg, 2008.

[8] K. Schlacher, M. Schoeberl, T. Rittenschober. Model based control of structures and machines, a dissipative and internal model based approach. 4^{th} *European Conference on Structural Control*, St. Petersburg, 2008.