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MATHEMATICAL MODELING OF MECHANICAL BEHAVIOUR OF LAYERED STRUCTURES USING A CONTINUUM APPROACH

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Abstract. Mechanical behaviour of a structure, consisting of rigid and soft layers, is considered by replacing the real structure with an equivalent uniform continuum. A brief representation of continuum models describing deformation of layered media, as well as some generalizations, is presented. The continuum models are compiled into three groups: classical models of effective elastic medium, models additionally accounting for bending of layers, and models suggesting that the bending prevails. As an example the problem of layered structure bending is addressed. The results obtained by various continuum models are compared with the results of direct FEM simulation accounting for individual layers. The FEM code, built for this purpose, incorporating various continuum media models and various types of elements is used. Estimation of applicability ranges for various continuum models is made. It is shown that disregarding the influence of layers bending can lead to significant errors. An example, deformation of a graphite plate, is considered.

1 Introduction. State of problem and research goals

For material consisting of a large number of layers the continuum approximation approach seems appropriate to describe its mechanical behaviour. In the framework of this approach the real structured media are replaced with a homogeneous continuum possessing some effective properties. In case of the perfect cohesion of layers such an equivalent homogeneous medium is just an anisotropic elastic one. The general solution of the problem of the effective elastic characteristics determination for such a medium is well known as obtained by Lifshitz and Rozentsveig [1]. However, if the medium allows relative sliding of the layers, then within the layers bending may occur at the places of significant stress gradients. The bending is accompanied by such effects as the violation of the shear stress parity rule and appearing of the moment stresses. A similar situation may take place even in the case of perfect cohesion between the layers, if one group of the layers is sufficiently compliant to be considered as slippery interfaces. The peculiarity distinguishing these cases from the anisotropic elasticity is the presence of the additional degree of freedom, associated with the relative movement of the layers. Various variants of models accounting for this effect were suggested by many researches starting with the pioneer work [2]. Some modifications and generalizations are suggested by the authors.

In the present work an example of multilayered plate deformation is considered in order to ascertain the range of parameters for which various types of continuum models are applicable.

2 Continuum models. Variational approach

Various types of continuum models were derived using a variational principle as follows. At the fine scale (of the order of layer thickness) a set of independent kinematic variables, such as displacements, are introduced and approximated by some set of parameters. Then using a variational principle the corresponding potential (energy) is derived in terms of the introduced independent variables. The equations of motion (equilibrium) and natural boundary conditions are obtained from this potential by the standard variation procedure. In the classical continuum mechanics, due to the assumption that there exists a limit of the energy of the medium at decreasing the volume element down to zero, the influence of high-order derivatives of independent field vanishes. For the structured media, such as layered media in question, this assumption is omitted (the representative volume element may not be less then element of the structure), and the set of the parameters, introduced to characterize the field within the volume element may include high-order derivatives of independent field and the additional degrees of freedom of the model.

Let us consider an application of the variational approach to obtaining a continuum model of layered media. Consider an area of the media possessing a cubic lattice structure and introduce the volume element ΔV, which is small comparing to the area dimensions, but not less then the size of the lattice structure. The undeformed element is shown in Fig.1. For simplicity, we restrict ourselves with 2-D case.

Generally, two types of deformation may be distinguished: the deformation of the lattice (Fig.2a) where the particles neighbouring before deformation remains neighbouring after deformation; and the relative movements of the neighbouring particles (Fig.2b). The first type of deformation will be called the elastic deformation, while the second one will be referred to as the quasiplastic deformation. The term "plastic" is used in this particular meaning, rather then implying energy dissipation.

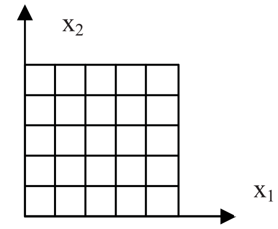

Figure 1. Undeformed element

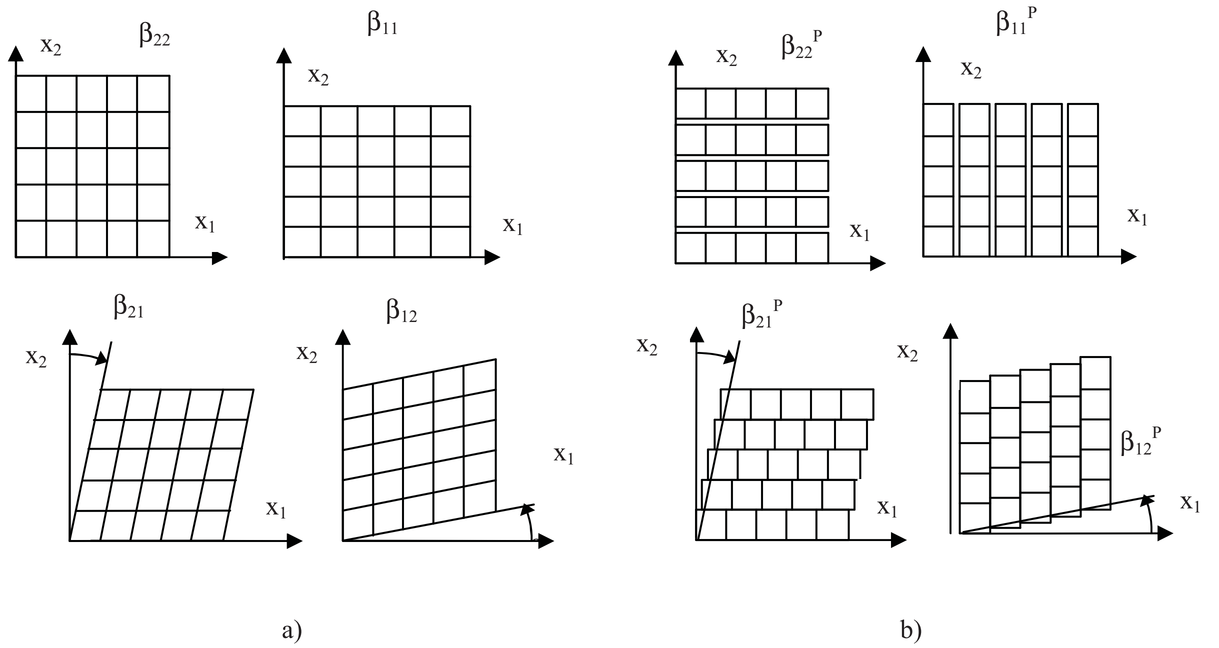

Figure 2. a) Homogeneous elastic distortion; b) Homogeneous quasiplastic distortion

Geometry of the deformed body may be described by the tensor of elastic distortion $\boldsymbol{\beta}$, with Cartesian components β_{ij}, and the components $\beta_{ij}{}^{P}$ of quasiplastic distortion. Geometries, corresponding to various components of elastic and plastic distortion, are shown in Fig.2a and Fig.2b. The sum of the elastic and plastic distortion is denoted the total distortion

$$\beta_{ij}{}^{T} \equiv U^{T}_{j,i} = \beta_{ij} + \beta_{ij}{}^{P} \tag{1}$$

Here U is the Cartesian displacement. Consider layered media with x_2-axis being normal to the layers. Such media may deform and relative sliding of the layers may occur, but not their separation. Hence, the only nonvanishing component of quasiplastic distortion is $\beta^{P}_{21} = \beta^{P} = \beta^{T} - \beta$.

In the framework of the classical mechanics of microstructured media [3], the energy density is supposed to be a function of total and plastic distortion and gradient of the elastic distortion. However, due to Eqv. (1), without loss of generality we may write it as a function of elastic and plastic distortion and the corresponding gradient

$$W = W\left(\beta_{ij}, \beta^{P}_{ij}, \beta_{ij,k}\right) \tag{2}$$

A particular expression for the potential of Eqv.(2) may be written in various ways. Thus, according to [4] (similar models were considered in [5-10]) it has the form

$$W = \frac{\lambda + 2\mu}{2}\left[U_{1,1}{}^{2} + U_{2,2}{}^{2}\right] + \lambda U_{1,1} U_{2,2} + \frac{\mu}{2}\left(U_{2,1} + U_{1,2} - \beta^{P}\right)^{2} + \frac{k}{2}\beta^{P2} + \frac{1}{2}\frac{D}{2h}\beta_{,1}{}^{2};\ D = \frac{8\left(\lambda + \mu\right)}{3\left(\lambda + 2\mu\right)}\mu h^{3} \tag{3}$$

The layers are assumed isotropic, λ and μ being Lame constants, 2h being the layer thickness. It is assumed that interlayer sliding is determined by the Wienkler-type law with the constant k. Here its first two terms correspond to tension and compression along and normally to layers, the third term corresponds to the shear within the layers, the forth correspond to the shear between the layers, the fifth correspond to bending. The standard procedure of variation of Eqv.(3) leads to equations of equilibrium and natural boundary conditions.

Using such a procedure various types of continuum models, both known and new, depending on particular type of Eqv.(2), were obtained, which we divide into three groups: classical models of effective elastic media (classical model, CM) [1], models of types [4-12] accounting for additionally bending of layers (universal models, UM), and models of types [13-14] supposing that bending prevails (bending models, BM). Also, some generalizations of the models were made [15].

In order to test the applicability of the continuum models and finding the ranges of their applicability the problem of deformation of the layered structure was addressed.

3 FEM modelling

The FEM code was built incorporating various continuum models and various types of elements, namely: 3-point simplex element, 6-point element, 3-point element with conjugated gradients. The last type of element seems to be used for the first time for the problems of such a kind (FEM modeling of the deformation of layered structure were made by [9, 10]).

To test the applicability of the continuum models and finding the ranges of their applicability the problem of deformation of the layered structure was addressed. Rectilinear area of the length L=340 and height H=31.5 consisting in 9 pairs of layers of thickness 3.5 were loaded along its long side with the pressure p=1, lateral sides were clamped; the interface was modeled by very thin (thickness being 0.01) soft layers. The other parameters are $\lambda = \mu = 1$ k $=10^{-5} \div 10^{2}$. All values of parameters are dimensionless. The result of FEM modeling using 3-point element with conjugated gradients are given in Fig.3 for the 300 element mesh (Fig.4). Calculations were also performed using meshes with 480, 700, 1400, 2800 elements. The results obtained by various continuum models using FEM simulation and simplified Ritz method were compared with the results of direct FEM simulation accounting for individual layers (three rows of elements of a same material properties for each layer, totally 10800 elements).

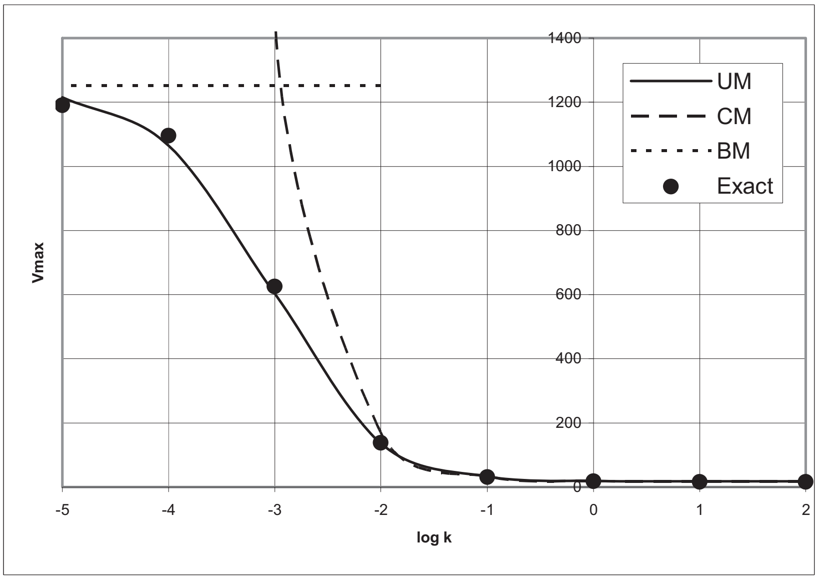

Figure 3. Results of calculation of maximal vertical displacement of the central point of the layered plate according to various models

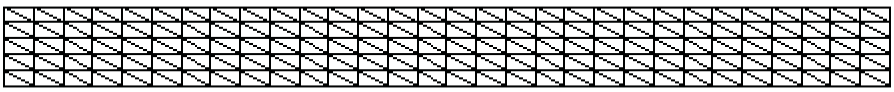

Figure 4. FEM net used for calculation

4 Main results and conclusions

For the addressed problem the ranges of applicability of three groups of models (CM, UM, BM) were obtained. They were determined by the parameter, characterizing relative rigidity of the layers k (see Fig.3). It was shown that CM is applicable for high and moderate relative rigidity of the soft layers (values of $k \geq 10^{-2}$); BM is applicable for extremely low relative rigidity of the soft layers (values of $k \leq 10^{-4} \div 10^{-5}$); UM is applicable for any value of rigidity of the soft layers. The results obtained by various variants of UM are very close to each other not allowing choosing the best of them.

The results obtained with the simplified Ritz method were also in a very good agreement with the direct FEM modeling. However the direct application of this approach is restricted with very simple geometries.

The comparison of various types of finite elements leads to the following conclusions. Using 3-point simplex element is inappropriate for these kinds of models, because it requires too many elements (comparable to the number of elements in direct modeling) for the good convergence to the exact solution. 6-point elements and 3-point element with conjugated gradients yield good convergence. Therefore, UM models yield good results with finite elements of high order (6-point element, 3-point element with conjugated gradients), which nevertheless gives an advantage in computing time.

An example of a graphite plate deformation is considered. It was shown that calculating deformations of small plates using CM could yield errors, and thus UM are preferable.

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