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CALIBRATION OF FLEXIBLE STRUCTURES FROM MULTIPLE MEASUREMENTS

(REDUNDANT MEASUREMENTS BETTER THAN COMPLEX MODELS)

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Abstract. The paper deals with calibration and identification of mechanical properties of mechanical structures of truss type. It includes both the structures of civil engineering constructions and the parallel kinematical structures of robots, machine tools and measuring machines. The common concept is that the mechanical structure is equipped with network of multiple sensors and based on the measurements the properties (models) of the investigated structure are reconstructed. The important problem is the selection of ratio between the number of reconstructed parameters and the number of measurements and/or the complexity of applied models. The common conclusion from both cases is that the usage of complex models should be replaced by redundant measurements for simpler models. The method of redundant measurement seems more robust and promising for industrial usage.

1 Introduction

The sensors are becoming miniaturized and cheap. This introduces the common concept that the mechanical structure is equipped with network of multiple sensors and based on the measurements the properties (models) of the investigated structure are reconstructed. The important problem is the selection of ratio between the number of reconstructed parameters and the number of measurements and/or the complexity of applied models. This paper deals with calibration and identification of mechanical properties of mechanical structures of truss type. It includes both the structures of civil engineering constructions and the parallel kinematical structures of robots, machine tools and measuring machines.

2 Identification of frame structure

One problem deals with the identification of structural stiffness model of a spatial frame structure (Fig. 1) using network of multiple sensors measuring the deformations of particular nodes (joints) for several loading cases. The necessary network of measurements can be significantly reduced. The structure treated as truss is more difficult to be identified than the frame structure. On the other hand the eliminated measurements in the inside nodes worsen the problem conditionality. The frame structure can be generalized to the complete identification of continuum model.

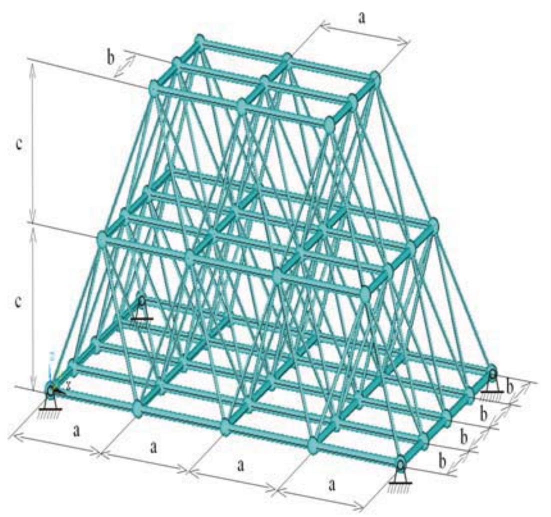

Figure 1. Spatial frame structure

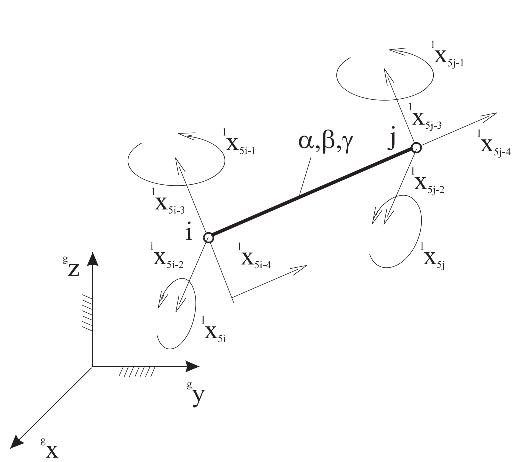

Figure 2. 3D beam element

2.1 Stiffness Reconstruction of 3D Frame

The goal was to apply the static non-destructive loads and measure displacements and rotations of the separate joints. From this measurement can be derived a complete set of the structural element stiffness parameters. For modeling of the particular members of the solved structure were used elements transferring axial forces and bending moment too.

The stiffness properties of the structure Fig. 2 can be described by the fundamental equation

$$ {}^{g}K\ u_j = f_j\,, \tag{1} $$

where f_j is a column vector of j-th applied load. K is a stiffness matrix of a solved structure in the global coordinate system and u_j is a column vector of the displacements and rotations of each joint for j-th applied load acquired from the network of sensors.

The stiffness matrix of a single member is expressed in the local coordinate system

$$ {}^{l}K_n = k_{n1}A_n + k_{n2}B_n + k_{n3}C_n\,, \tag{2} $$

where k_{n1} is the unknown stiffness parameter corresponding to the tension or compression of the member, k_{n2} is the unknown stiffness parameter corresponding to the bending according to a bending moment about the neutral axis y of the local coordinate system and k_{n3} is the unknown stiffness parameter corresponding to the bending according to a bending moment about the neutral axis z of the local coordinate system. The matrices A_n, B_n and C_n are constant matrices that can be derived from the FE formulation of the beam element. Transforming eq. (2) to the global coordinate frame and reformulating for unknown stiffness parameters can be acquired over-constrained system of the linear equations.

$$ \begin{bmatrix} A_1 \\ \dots \\ A_j \end{bmatrix} k = \begin{bmatrix} f_1 \\ \dots \\ f_j \end{bmatrix}, \tag{3} $$

This is a system of linear overdetermined equation. They can be solved and all unknown stiffness parameters can be computed from eq. (3) by pseudoinverse but better solution is using Singular Value Decomposition method [2].

2.2 Reduced Redundant Measurements

Disadvantage of the presented approach is the necessity to obtain the vector of deformation in direction of particular degrees of freedom (DOFs). Thus a question arises whether the number of measuring points on the structure can be reduced. From the method of static condensation can be derived the system of nonlinear equations

$$ f\left(k_{n1}, k_{n2}, k_{n3}\right) = 0\,. \tag{4} $$

The solution of these equations is a very time consuming. Therefore it was tested the direct iterative evaluation of the stiffness parameters.

The static “Guyan’s” condensation method can be reformulated to

$$ \begin{aligned} &\left(\sum_{n=1}^{Nelem} k_{n,1}\left({}^{g*}\mathrm{K}_{\mathrm{mm}}\right)_{n,1}\right)\mathrm{u_m} + \left(\sum_{n=1}^{Nelem} k_{n,2}\left({}^{g*}\mathrm{K}_{\mathrm{mm}}\right)_{n,2}\right)\mathrm{u_m} + \left(\sum_{n=1}^{Nelem} k_{n,3}\left({}^{g*}\mathrm{K}_{\mathrm{mm}}\right)_{n,3}\right)\mathrm{u_m} + \\ &\left(\sum_{n=1}^{Nelem} k_{n,1}\left({}^{g*}\mathrm{K}_{\mathrm{ms}}\right)_{n,1}\right)\mathbf{Z}_{sm}\mathrm{u_m} + \left(\sum_{n=1}^{Nelem} k_{n,2}\left({}^{g*}\mathrm{K}_{\mathrm{ms}}\right)_{n,2}\right)\mathbf{Z}_{sm}\mathrm{u_m} + \left(\sum_{n=1}^{Nelem} k_{n,3}\left({}^{g*}\mathrm{K}_{\mathrm{ms}}\right)_{n,3}\right)\mathbf{Z}_{sm}\mathrm{u_m} = \mathrm{f_m} \end{aligned}, \tag{5} $$

where u_m is vector of the master DOFs and matrix Z_{sm} is free of the unknown stiffness parameters. The index m means master and s means slave. From this equation can be by the iterative process computed the unknown stiffness parameters of the solved structure. It is necessary just to estimate the ratio of these parameters.

By this way it can be eliminated a lot of the measured deformations. It is always possible to reduce the fully redundant measurement to about 50%. Currently investigated hypothesis is the possibility to eliminate all inner measurements and to measure just the boundary elements and use only 3 loading cases.

3 Determination of TCP position of compliant PKM

Another problem deals with the determination of the TCP position of machine tool accurately during the operation of the machine tool in case of parallel kinematic machines (PKM) (Fig. 3) that is treated as compliant mechanism. The procedures for the determination of TCP are described for PKM Sliding Star. Sliding Star is a functional model of machine tool with parallel kinematical structure (Fig. 3). It is redundantly actuated mechanism, i.e. it has more drives than the degrees of freedom. Sliding Star has 3 degrees of freedom and 4 drives (moving screws).

Two approaches for the determination of TCP position of deformable PKM can be distinguished. The first one is based on the model of compliant mechanism of PKM. The second one replaces this complex model by redundant measurement. The first approach uses relatively complex model of compliant mechanism and is based on the belief that simulation can truly predict the reality. The second approach uses just the geometrical model where the flexibility is described by the variability of mechanism dimensions. This approach is based on extensive redundant measurement.

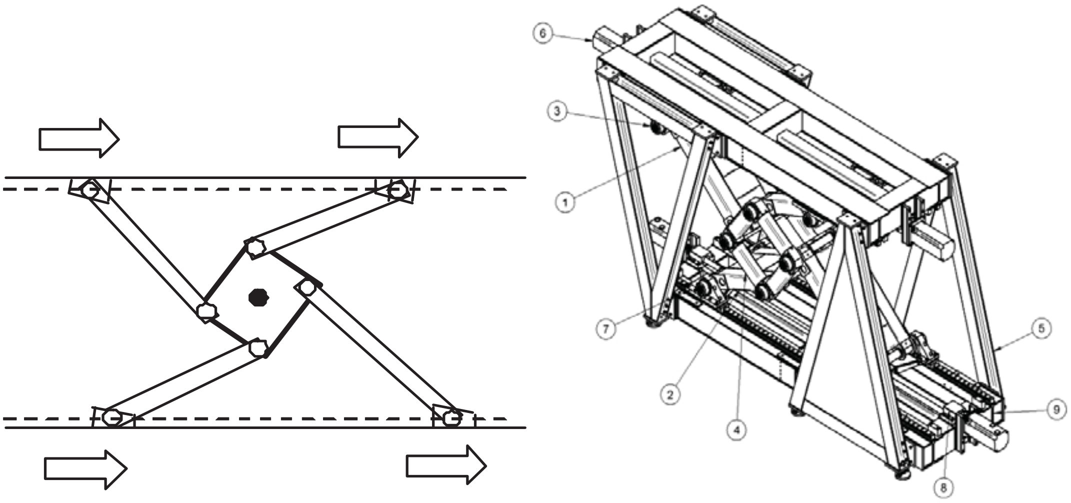

Figure 3. Parallel kinematic machine Sliding Star

3.1 Method of compliant model

The computational model of compliant mechanism of Sliding Star consists of force (or deformation) model in Fig. 4 and of geometrical model in Fig. 5. The usually considered measurement scheme is in Fig. 6a, i.e. the measurement of 4 positions of sliders. It consists just from the sensors of drives of moving screws. As the variables it is necessary to introduce the lengths of arms L1, L2, L3, L4, the cartesian coordinates of points A, B, C, D xA, xB, xC, xD, yA, yB, yC, yD, the arm angles $\alpha,\beta,\gamma,\delta$ and the platform turning ε, the stiffnesses of all elements, the acting forces of drives, cutting and the reaction forces.

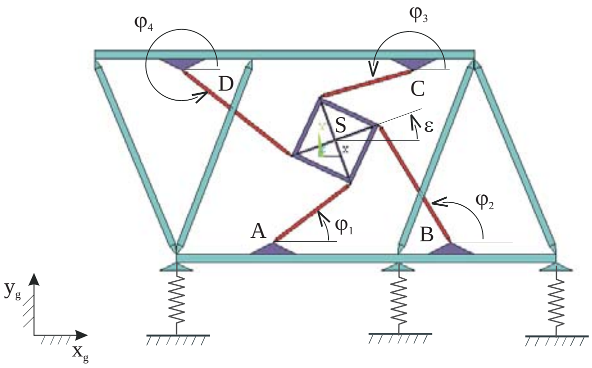

Figure 4. Force model of Sliding Star

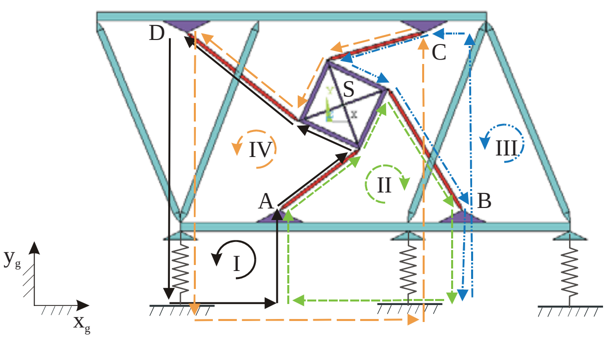

Figure 5. Geometrical model of Sliding Star

For simplicity the platform described by the dimension h is supposed to be completely rigid. From the geometrical model the equations of closure of kinematical loops are assembled, for example for the loop AEFB

$$
\begin{aligned}
x_A + (L_1 + \Delta L_1)\cos(\alpha) + h\cos(\varepsilon + \frac{\pi}{4}) &= x_B + (L_2 + \Delta L_2)\cos(\beta) \\
y_A + (L_1 + \Delta L_1)\sin(\alpha) + h\sin(\varepsilon + \frac{\pi}{4}) &= y_B + (L_2 + \Delta L_2)\sin(\beta)
\end{aligned}
\qquad (6)
$$

It is possible to assemble 6 such equations. From the force model the equations for the solution of truss are assembled, for example for the node A

$$\begin{aligned} x: &\quad F_1 + k_1 \Delta L_1 \cos(\alpha) = 0 \\ y: &\quad k_{yB} y_B + k_1 \Delta L_1 \sin(\alpha) = 0 \end{aligned} \tag{7}$$

It is possible to assemble 16 such equations. These equations 22 equations are solved for the 22 unknowns ΔL_1, ΔL_2, ΔL_3, ΔL_4, ε, α, β, γ, δ, reactions R_{yA}, R_{yB}, R_{yC}, R_{yD}, in the sliders, the axial forces S_1, S_2, S_3, S_4, S_5, in the rigid truss of the platform, the loading forces from the cutting acting on the platform Fx, Fy, Mz, and possible also the redundant position x_D. However, this approach suffers from the difficulty of precise modeling of nonlinear effects like friction forces, cutting forces, deformation models, etc.

3.2 Method of redundant measurements

The computational model of TCP determination by redundant measurement consists just of the geometrical model in Fig. 5. The measurement scheme must be more redundant. It is measured the displacement of all drives (sliders) x_A, x_B, x_C, x_D, the rotation of arms φ_1, φ_2, φ_3, φ_4 and the vertical deformation of the frame at the position of sliders y_A, y_B, y_C, y_D (Fig. 6b). In this case only the equations from the geometrical model are used. For example for loop II can be written

$$\begin{aligned} x_A + LL_1\cos(\varphi_1) + h\cos(\varepsilon + \frac{\pi}{4}) = x_B + LL_2\cos(\varphi_2) \\ y_A + LL_1\sin(\varphi_1) + h\sin(\varepsilon + \frac{\pi}{4}) = y_B + LL_2\sin(\varphi_2) \end{aligned} \tag{8}$$

where LL_1 describes the unknown deformed length of the arm L1. It is possible to assemble 6 such equations and these 6 equations are solved for 5 unknowns LL_1, LL_2, LL_3, LL_4, ε, where two of the measurements x_D, y_D, φ_4 are redundant. The redundancy is however only helpful for the solution of the equations.

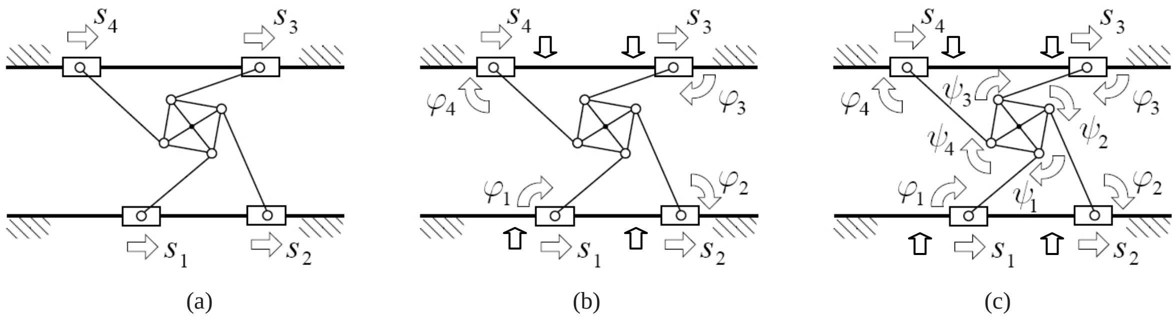

Figure 6. Different measurement schemes for Sliding Star

In case of the flexibility of the whole PKM including the platform the redundant measurement scheme must be extended (for example Fig. 6c).

In general the complex compliant model with difficult nonlinearities like unknown friction is replaced by simpler geometrical model with redundant measurement that replaces the missing knowledge of parameters.

4 Conclusion

The common conclusion from both cases is that the usage of complex models should be replaced by redundant measurements for simpler models. The method of redundant measurement seems more robust and promising for industrial usage.

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5 References

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