

# MODEL DRIVEN CONCEIVED / DATA DRIVEN TUNED CONTROL SYSTEMS

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**Abstract.** Three approaches for control system design are faced on this paper and the role played by the plant model highlighted. On one side we have the well known Model-Based Control (MBC), that is further extended to the Model-Driven Control (MDC) conception of a control system and, finally, embedded within the Data-Driven Controller (DDC) approach for design. The advantages of conceiving the overall control system as a MDC are raised and the possibility of doing the design of the controller as a DDC overcomes the sometimes imposed constraint of suitable low complexity (linear) models.

## 1 Introduction

Model-Based Control (MBC) is a well known and accepted approach for the conception and design of feedback control systems. The disposal of a model as a meaningful representation of the real world (in fact the plant we are to control), allows to simulate, predict and design a suitable controller to determine its behavior. This is the route taken by modern control approaches where a controller is computed on the basis of the plant model. However, the finally deployed controller, even computed on the basis of a plant model does not explicitly contains the plant model. In this sense, the approach is *model-based*. On the other hand, within a Model-Driven Control (MDC) formulation the plant model is explicitly used as a part of the controller. Perhaps the more well known example of MDC architecture is the *Internal Model Control* (IMC) proposed by [1]. This structure has some interesting features, all of these are extensively discussed in [1]. The main drawback of the IMC formulation is the need for the plant to be stable, in [2] a general formulation was proposed where no need for open-loop stability is required. The resulting structure is also model-driven and several advantages of the explicit use of the model on the controller feedforward components have been recently reported. See [3] and [4].

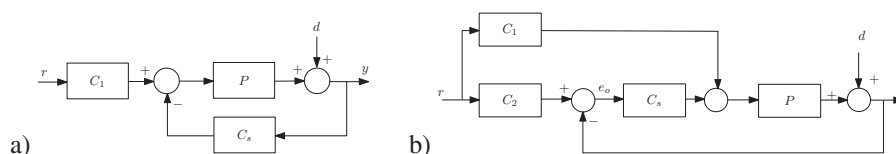
From a completely different perspective, during the last decade, the relationship between system modelization and system control has been analyzed. Nowadays, it has become clear that the goodness of a model is dependent of how useful it becomes in the control design task (see introduction in [5]). This concept leads to a broad classification of control synthesis approaches as: Model-based Control and Data-based Control. Data-based, or Data-Driven controllers (DDC) refers to the family of methods which finds the controller directly from data and never attempts to determine a model.

What is presented in this paper is the conjunction of the advantages of MDC system conception but by using a DDC approach for tuning. Among other advantages it is seen that what really matters is the inclusion of the plant information into the controller. The controller components that are structured on the basis of the plant model and that do operate in feedforward mode, are designed under a DDC approach. Therefore directly translating the available plant information to the (explicitly) model dependent controller terms. Several advantages with respect to the other existing DDC formulations (but acting on a MBC) are highlighted.

## 2 The Control Problems

From now on, in this paper, the control problems will be formulated on a Two-Degrees of Freedom (2DoF) structure in order to achieve the desired reference tracking and disturbance rejection behaviors in the discrete time domain. Also all the controllers are considered to be of restricted complexity. How to measure the complexity of a controller is not clear at all. This is a topic that deserve more attention. As far as this paper is concerned, when saying that a controller has restricted complexity, it means that the number of parameters is limited. The standard 2DoF structure is shown on Fig. 1 a)

The inputs-output relationship is:



**Figure 1:** Standard and Alternate 2DoF structures

$$y = \frac{PC_1}{1+PC_s}r + \frac{1}{1+PC_s}d \quad (1)$$

Controller  $C_1$  is a prefilter that is intended to compensate what  $C_s$  can't achieve in the reference-tracking problem. No independence between the tracking performance and the disturbance-rejection is achieved, since a change in  $C_s$  affects both problems.

Being  $M(z)$  the desired reference-to-output transfer function and  $S(z)$  the desired Sensitivity Function. In general, the control problem can be stated as finding the parameters  $[\theta_r, \theta_s]$  that minimize the control criterion (see [[6]]):

$$J_{standard}(\theta_r, \theta_s) = \left\| \left( \frac{P(z)C_1(z; \theta_r)}{1+P(z)C_s(z; \theta_s)} - M(z) \right) W_M(z) \right\|_2^2 + \left\| \left( \frac{1}{1+P(z)C_s(z; \theta_s)} - S(z) \right) W_S(z) \right\|_2^2 \quad (2)$$

Obviously solving the minimization of (2) implies some knowledge on the plant or having a model of it. If during the determination of the controller, knowledge of the plant is used, but the controllers don't have the explicit use of the model of the plant within, the problem is said to be solved by MBC. For example in [6] the information of the plant is used in the form of pure data taken from an experiment on the plant (not a model of it). The minimization that is carried out in [6] to find the parameter of the controllers try to minimize (2) but, the controllers are given no direct relation with the plant model other than minimize (2).

On the other hand, the controllers  $C_1$  and  $C_2$  of the structure shown on Fig. 1b) have an interesting conception that takes into account the plant model. If the plant model is factorized as  $P = ND^{-1}$  with  $N$  and  $D$  stable and proper transfer functions. It can be found that if  $C_1 = NQ$  and  $C_2 = DQ$ , the input-output transfer function became  $y = NQr$ . Of course, this relation should be equal to  $M$ , the desired reference to output transfer function, this leads to

$$Q = N^{-1}M \quad (3)$$

The feedback controller  $C_s$  only acts, if relation (3) can't be fulfilled completely or in case of disturbance at the output of the plant. In other words,  $C_s$  is responsible of the stability of the closed-loop system and the disturbance rejection performance, which can be treated as a separate sensitivity shaping problem. In this case, the problem is resolved as a MDC, since the controllers explicitly contains the model of the plant. The prefilter block ( $C_1$ ) should represent the target reference to output transfer function, so, one can just define  $C_1 = M$ . Then the feedforward controller ( $C_2$ ) have to provide the correct control input to the plant to achieve the tracking objective. Since this controller have to be determined, let's re-name it  $C_{ff}$ . The feedback controller  $C_s$  continues to be the responsible of the closed-loop, so its meaning remains the same. In this case, the control problem can be formulated to minimize the following objective function:

$$J_{alternate}(\theta_{ff}, \theta_s) = \left\| \left( \frac{PC_s(z; \theta_s)}{1+PC_s(z; \theta_s)} \left( \frac{C_{ff}(z; \theta_{ff})}{C_s(z; \theta_s)} + M \right) - M \right) W_M(z) \right\|_2^2 + \left\| \left( \frac{1}{1+PC_s(z; \theta_s)} - S \right) W_S(z) \right\|_2^2 \quad (4)$$

However, it is possible to use the Virtual Reference Feedback Control (VRFT) framework of [7], [8] and [6] to find the controllers parameters in this structure that minimize (4). It is possible to *identify* the  $C_{ff}$  controller from the input/output data. It is desirable to find the best approximation of  $Q$  as given by (3), or as in this case, the best approximation of  $MP^{-1}$  since ideally,  $C_{ff} = MP^{-1}$ . If the signal  $\bar{r} = M^{-1}y$  is introduced in the feedforward controller, the output should be  $u$ . The direct consequence of this fact is that, one can use an identification method or an optimization method to determine  $C_{ff}$ , using  $\bar{r}$  (the filtered version of  $y$  through  $M^{-1}$ ) as the input, and  $u$  as the output values.

### 3 Advantage of the Model Driven Conceived / Data Driven Tuned Controller

The main advantage of the structure in Fig. 1 is the separation between the tracking and the disturbance rejection problems. The ideal controllers (that is, the ones that would make  $J_{alternate} = 0$ )  $C_{ff0}$  and  $C_{s0}$  for this alternate structure, are given by

$$C_{ff0} = P^{-1}M, \quad C_{s0} = \frac{1-S}{SP} \quad (5)$$

By introducing (5) into (4) it can be shown that it reduces to (6), where the "tracking" problem and the "disturbance rejection" are separated (dropping the  $z$  argument).

$$J_{alternate}(\theta_{ff}, \theta_s) = \left\| \frac{P(C_{ff} - C_{ff0})}{1 + PC_s} \right\|_2^2 + \left\| \frac{P(C_{s0} - C_s)}{(1 + PC_s)(1 + PC_{s0})} \right\|_2^2 \quad (6)$$

As the problems are separated, it's possible to optimize each controller for the specific task it is intended to deal with ( $C_{ff}$  for tracking and  $C_s$  for disturbance rejection). This “extra” freedom, helps to improve the overall performance of the system, given a good approximation of (3) in the sense that there is no compromise between improving the tracking and the disturbance rejection.

A simulation example is carried out by using the same plant as in the example in [6]. The plant is given by

$$P(z) = \frac{0.1622z^{-1} - 0.01622z^{-2}}{1 - 1.7z^{-1} + 0.8825z^{-2}} \quad (7)$$

The sampling period is assume to be 1s. The target transfer function are given by

$$M(z) = \frac{(1 - \alpha)z^{-1}}{1 - \alpha z^{-1}}, \quad S(z) = 1 - \frac{(1 - \beta)z^{-1}}{1 - \beta z^{-1}} \quad (8)$$

Note that depending on the value of  $\alpha$  and  $\beta$  we may have  $S + M \neq 1$

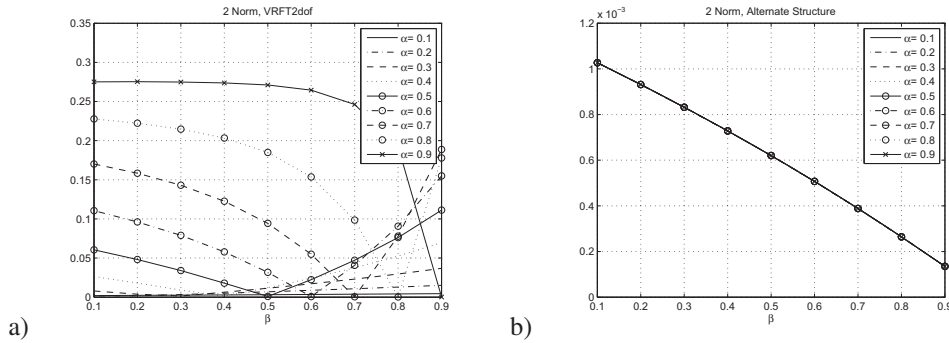
The controllers  $C_1(z, \theta_s)$  and  $C_s(z, \theta_s)$  have the same structure as in [6] and given by

$$C(z; \theta) = \frac{\theta_0 + \theta_1 z^{-1} + \theta_2 z^{-2} + \theta_3 z^{-3} + \theta_4 z^{-4}}{1 - z^{-1}} \quad (9)$$

$C_{ff}$  was found using the Output Error (OE) identification method ([9]). The structure chosen was

$$C_{ff}(z, \theta_{ff}) = \frac{\theta_{b0} + \theta_{b1} z^{-1} + \theta_{b2} z^{-2}}{1 + \theta_{a1} z^{-1} + \theta_{a2} z^{-2}} \quad (10)$$

It has the same number of parameters, but they appear in both the numerator and denominator. The independence between reference and tracking is shown when varying parameters  $\alpha$  and  $\beta$  in figure 2. In this example, the OE method achieves the ideal  $C_{ff}$  so both problems are completely decoupled. That's why, in Fig. 2b) the value of  $J_{alternate}$  doesn't change when varying  $\alpha$



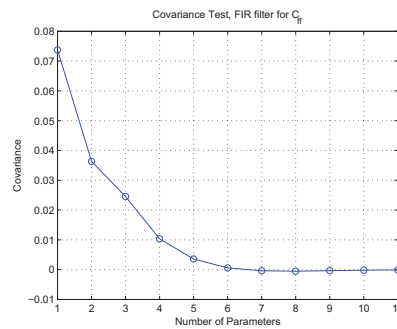
**Figure 2:** Two norm of cost function  $J_{standard}$  for the two-degrees of freedom VRFT (a) and of cost function  $J_{alternate}$  for the alternate structure (b)

Another important advantage is the use of the signal  $e_o$  as a measure of how well the structure of  $C_{ff}$  has been chosen. The signal  $e_o$  in Fig.1b) represent the difference between the desired and the achieved input-to-output relationship. If the virtual reference  $\bar{r}$  were applied to the system, then the *virtual error*  $\bar{e}_o$  would be given by

$$\bar{e}_o = M\bar{r} - (C_{ff} + MC_s) \frac{P}{1 + C_s P} \bar{r} \quad (11)$$

Knowing that  $\bar{r} = M^{-1}y$  and making the approximation  $\frac{C_s P}{1 + C_s P} \approx 1 - S$  The virtual error can be approximated by

$$\bar{e}_o = \left( S - \frac{C_{ff}}{MC_s} (1 - S) \right) y \quad (12)$$



**Figure 3:** Covariance Test, for a FIR structure for  $C_{ff}$

One can know the wellness of the structure of  $C_{ff}$  because, if there is a correlation between the signal  $\bar{r}$  and  $\bar{e}_o$ , the structure chosen for  $C_{ff}$  is not appropriate. In the alternate structure, when controller  $C_2 = M$ , the error should be zero, if  $C_1 = C_{ffo} = MP^{-1}$ . If there is a correlation between the signals, it means that the controller  $C_{ff}$  found is not adequate. To check the results of the covariance test, the  $C_{ff}$  structure will be linear-in-the-parameters (that is  $C_{ff} = \beta(z)\theta_{ff}$ ), so a standard least squares problem is obtained. While maintaining  $C_s$  constant. The choice of functions  $\beta(z)$  is important to define the number of parameters needed. Because of space restrains, only will be shown the results for the controller  $C_{ff}(z; \theta_{ff})$  being a FIR filter. Using the same plant as above and with  $\alpha = 0.5$  and  $\beta = 0.1$ , the covariance test between  $\bar{r}$  and  $\bar{e}_o$  is performed for different number of parameters. The result is shown in Fig. 3. As it can be seen, after 8 parameters, the variation in the covariance is negligible, so, for this particular example, the number of parameter needed for this structure is 8.

## 4 Conclusions

The use of the MDC inside a VRFT framework was shown to have some advantages over a standard 2DoF structure. The most important is that the tracking reference and the disturbance rejection problems become decoupled. This allows the designer to choose different target transfer functions for each problem without a compromise between the two problems. Also is important to notice that, in order to find each controller, is not necessary to use the same methods (for example, controller  $C_{ff}$  was found either using an OE method or simply a least squares optimization, no matter how  $C_s$  was found). The search for a way to choose the structure of the controller from data is an interesting open area (from the knowledge of the authors). In this case, the structure was supposed to be restricted, anyway, there is no rule to say a priori, how many parameters are enough for the controllers. The covariance test proposed in this paper, could be a start in order to determine a method to choose the number of the parameter of the controller from the data itself. More of this is currently carried out by the authors.

## 5 Acknowledgements

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