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MODELING AND NONLINEAR CONTROL OF MAGNETIC LEVITATION SYSTEMS WITH HYSTERESIS

N. Özdemir[1], B.B. İskender[1]
[1]Balıkesir Univ., Balıkesir, Turkey

Corresponding author: N. Özdemir, Balıkesir University, Department of Science and Arts, Cagis Campus, 10145 Balıkesir, Turkey; nozdemir@balikesir.edu.tr

Abstract. In this work, a magnetic levitation system is modeled with its hysteresis characteristic and a nonlinear controller is developed that the system output tracks a desired reference input. Hysteresis nonlinearity of the system which arises from ferromagnetic material in electromagnet is modeled by using Duhem model. An equivalent linear system is obtained by a nonlinear controller and the linear system is stabilized with pole placement. The simulation results show the system converges to the desired reference input.

1 Introduction

Magnetic levitation systems are nonlinear and hysteretic systems which have many application area such as high speed trains, frictionless bearing and vibration isolation system, see [1, 2]. Control problem of these systems is very important since magnetic levitation systems are nonlinear and open-loop unstable. Therefore, a lot of works have been reported to control magnetic levitation system in recent years. Most of these researches are based on linearized model around an operating point, see [3, 4]. However, deviating from operating point can cause deterioration of system behavior. Therefore, it is necessary to design nonlinear controller for a large domain of stability. Besides, the losing of energy due to hysteretic feature of the system, such as the heating of electromagnet or the change of inductance should be considered.

Feedback linearization is commonly used method to developed nonlinear controller for magnetic levitation systems, see [8, 9, 10]. Furthermore, sliding mode control is a nonlinear controller design approach for these kinds of systems, see [5, 6, 7]. However, these works ignored hysteresis effects in magnetic levitation systems. So far, very little work has been published that controls magnetic levitation system with effecting of hysteresis feature. For example, magnetic levitation system has been modeled with Preisach model of hysteresis and the inverse of hysteresis has been applied to the system for its compensation in [11]. Moreover, magnetic levitation system has been modeled with input backlash, and the smooth adaptive inverse of backlash has been applied to controller design with backstepping approach in [12].

In this paper, the mathematical model of a magnetic levitation system is given by considering its hysteresis feature. Duhem model is used to model hysteresis relation between the magnetic field and the magnetic induction of the system. The obtained system is nonsmooth because of hysteresis is a nonsmooth nonlinearity. Therefore, a derivative definition of Duhem hysteresis is given with its Lipschitz property. Then the system is controlled with feedback linearization method which is combined with pole placement technique. Finally, simulation results of the system are given and it is shown that the system tracks the desired reference input.

2 Modeling of Magnetic Levitation Systems with Hysteresis

In this section, mathematical model of magnetic levitation system is given. Figure 1 demonstrates the working principle of magnetic levitation systems. The system consists of an electromagnet and a ferromagnetic ball. The ball is levitated by a magnetic force which is produced by the electromagnet. Control purpose of the system is to tune voltage to reach desired equilibrium position.

The electrical component of the system is

$$u(t) = Ri(t) + L\frac{di(t)}{dt}, \tag{1}$$

where u is the controlled voltage applied to the electromagnet, i is produced current, R is coil resistance and L is inductance of the electromagnet. The mechanical component of the system is

$$m\frac{d^2y(t)}{dt^2} = F_m - mg, \tag{2}$$

where m is mass of the ball, g is gravity acceleration, y is the distance between the ball and the electromagnet, and F_m is the magnetic force. The electromagnetic force is

$$F_m = \frac{A}{2\mu_0}\left(\frac{B}{y}\right)^2, \tag{3}$$

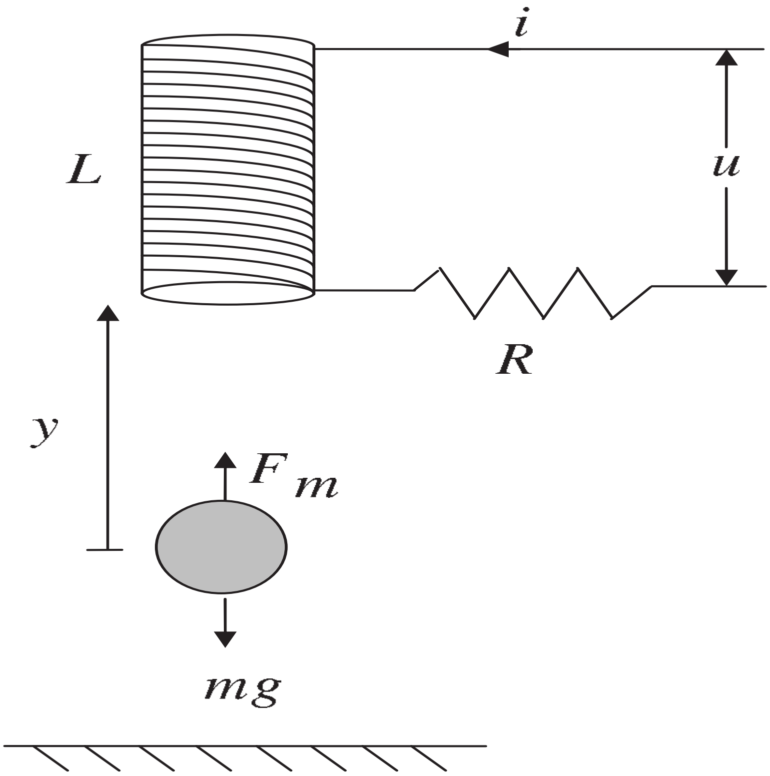

Figure 1: Magnetic levitation system

where A is sectional area of the electromagnet, μ_0 is the magnetic permeability of air, B is inductance of the electromagnet which depends on magnetic field H. The magnetic field is calculated by current i and turn number of the coil N;

$$H = Ni. \tag{4}$$

Relation between B and H is a hysteresis which is denoted by Φ

$$B = \mu_0 \Phi(H). \tag{5}$$

Substituting (5) to (3) we obtain

$$F_m = K\left(\frac{\Phi(H)}{y}\right)^2, \quad K = \frac{A\mu_0}{2}. \tag{6}$$

2.1 Duhem Model of Hysteresis

Hysteresis is a nonlinear relation between input and output functions which can be mathematically represented by causal and rate independent operator Φ. There are different kinds of hysteresis models, for example Relay, Stop, Play, Duhem, Preisach and Prandtl which are commonly used hysteresis models in the literature. In this work, hysteresis properties of the system is defined by Duhem model which is efficient to model electromagnetic hysteresis, see [13, 14, 15, 16]. Duhem model is based on the property that the output of the model changes its character when the input changes its direction, see [17]. This model is defined by the following differential equation

$$\frac{dB}{dt} = \alpha\left[\zeta(H) - B\right]\left|\frac{dH}{dt}\right| + \eta(H)\frac{dH}{dt}, \tag{7}$$

where α is a constant which gives hysteresis width, $\zeta(\cdot)$ and $\eta(\cdot)$ are some functions that satisfy given condition in [13, 14]. Duhem hysteresis is Lipschitz continuous model and it is differentiable except the derivative of its input is zero, see [18].

3 Nonlinear Control Design of Magnetic Levitation Systems

Feedback linearization is a method that transforms original system into a simpler equivalent system. It is usually used for obtaining a linear or partially linear system to implement linear control law, see [19]. In this work, the

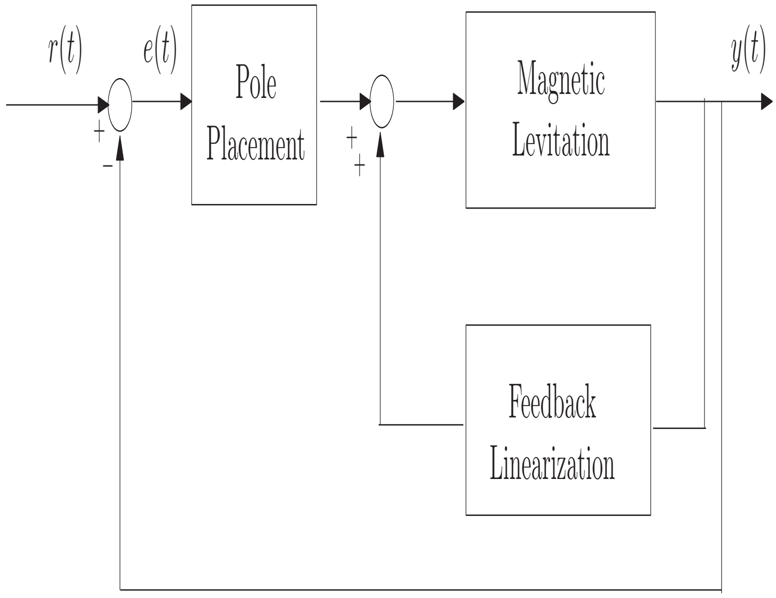

Figure 2: Block diagram of controller design

system is controlled by feedback linearization technique combined with pole placement. Block diagram of the controlled system is given in Figure 2.

Using the following change of variables

$$x_1 = H,\ x_2 = y,\ x_3 = \dot{y}, \tag{8}$$

the state space presentation is obtained as

$$\dot{x}_1 = -\frac{R}{L}x_1 + \frac{N}{L}u,\ \dot{x}_2 = x_3,\ \dot{x}_3 = \frac{K}{m}\left(\frac{\Phi(x_1)}{x_2}\right)^2 - g. \tag{9}$$

The output of the system y is position of the ball, so we can write the system in the following format:

$$\dot{x} = f(x) + g(x)u,\ y = h(x), \tag{10}$$

where

$$f(x) = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}^T = \begin{bmatrix} -\frac{R}{L}x_1 \\ x_3 \\ \frac{K}{m}\left(\frac{\Phi(x_1)}{x_2}\right)^2 - g \end{bmatrix}, \tag{11}$$

$$g(x) = \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix}^T = \begin{bmatrix} \frac{N}{L} \\ 0 \\ 0 \end{bmatrix}, \tag{12}$$

$$h(x) = x_2 \tag{13}$$

are functions of $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ (T denotes transpose of the matrix). To apply feedback linearization method f, g and h functions have to be differentiable. However, in (11) f is not differentiable because of hysteresis nonlinearity. Using Lipschitz continuity of Duhem model, we can define derivative of the model as

$$\frac{d\Phi(x_1)}{dx_1} = \begin{cases} \frac{d\Phi(x_1)}{dx_1}, & for\ \frac{dx_1}{dt} \neq 0, \\ \lambda, & for\ \frac{dx_1}{dt} = 0, \end{cases} \tag{14}$$

where λ is Lipschitz constant.

For feedback linearization of the system, the following coordinate transformation is applied;

$$z = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}^T = \begin{bmatrix} h(x) \\ L_f h(x) \\ L_f^2 h(x) \end{bmatrix}, \tag{15}$$

where $L_f h(x)$ is Lie derivative of $h(x)$ with respect to the vector field $f(x)$ which is defined by

$$L_f h(x) = \sum_{i=1}^{3} \frac{\partial h(x)}{\partial x_i} f_i(x) \text{ and } L_f^2 h(x) = L_f \left(L_f h(x) \right). \tag{16}$$

After some calculation

$$z = \begin{bmatrix} x_2 \\ x_3 \\ \frac{K}{m}\left(\frac{\Phi(x_1)}{x_2}\right)^2 - g \end{bmatrix} \tag{17}$$

is obtained and the System (10) can be rewritten in terms of new coordinates as

$$\dot{z} = \begin{bmatrix} L_f h(x) \\ L_f^2 h(x) \\ L_f^3 h(x) + L_g L_f^2 h(x) u \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ a(x) + b(x) u \end{bmatrix}, \tag{18}$$

where

$$a(x) = -\frac{2KR}{mL}\Phi(x_1)\frac{d\Phi(x_1)}{dx_1}\frac{x_1}{x_2^2} - \frac{2K}{m}\Phi(x_1)^2\frac{x_3}{x_2^3} \tag{19}$$

and

$$b(x) = \frac{2KN}{mL}\Phi(x_1)\frac{d\Phi(x_1)}{dx_1}. \tag{20}$$

The nonlinear controller is chosen to eliminate the nonlinear terms of the System (18) as the following

$$u = \frac{v - a(x)}{b(x)}, \tag{21}$$

where v is a controller for the linearized system. Substituting (21) into (18), the linearized system is obtained which is in the form of Brunovsky controller as

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v, \tag{22}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} z. \tag{23}$$

(22) is unstable because of multiple roots at the origin. To achieve stability of the System (22), the linear controller is designed for pole placement. For reference function $r(t)$, the linear controller is chosen as in the following

$$v = -k_1 (r(t) - z_1) + k_2 z_2 + k_3 z_3, \tag{24}$$

where k_1, k_2, k_3 are coefficients of characteristic equation of the system. Substituting (24) into (22) the following is obtained:

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ k_1 & k_2 & k_3 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ -k_1 \end{bmatrix} r, \tag{25}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} z. \tag{26}$$

k_1, k_2, k_3 are parameters to be chosen such that characteristic polynomial

$$q(s) = s^3 - k_3 s^2 - k_2 s - k_1 \tag{27}$$

is a Hurwitz polynomial. Hence, the System (25) is stable.

4 Simulation Results

In this section, simulation results of the magnetic levitation system are presented for a sinusoidal reference input $r(t) = 4.5\sin(2.3t)$. For Duhem model of hysteresis, $\alpha = 1$; $\zeta(H) = 3.1635H$ and $\eta(H) = 0.345$ are chosen. Simulation results are presented for $k_1 = -10^9, k_2 = -3.10^6, k_3 = -3.10^2$ so that the roots of the System (25) are $p_1 = p_2 = p_3 = -1000$. The other system parameters are given in the following table.

Parameter	Presentation	Value	Unit
mass	m	0.011	kg
coil resistance	R	52	Ω
coil inductance	L	1.227	H
coil turns	N	1200	turns
maximum voltage	$u_{\max}$	24	V
minimum voltage	$u_{\min}$	-24	V
sectional area	A	4.91×10^{-4}	m^2
magnetic permeability of air	μ_0	$4\pi \times 10^{-7}$	H/m
gravity acceleration	g	9.81	m/s^2
initial ball position	y_0	0.005	m
initial current	i_0	0.01	A

Figure 3 shows that the output of the system is converged to reference input function which means the control purpose is achieved. Figure 4 depicts the controller output. The magnetic field H and the magnetic induction B are given in Figure 5 and Figure 6, respectively. Finally, Figure 7 describes the magnetic field H against to the magnetic induction B which displays hysteresis behavior of the controlled System (25).

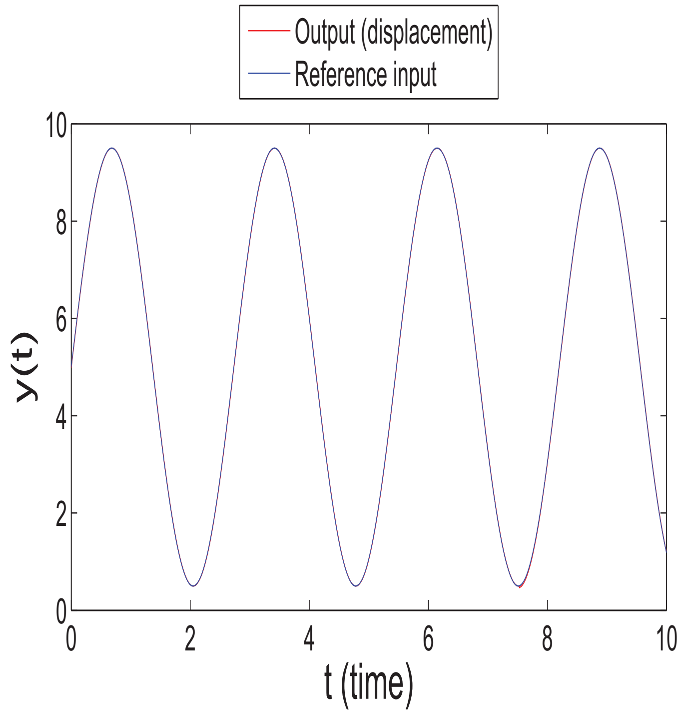

Figure 3: Comparison of the system output with respect to reference input

5 Conclusions

In this work, hysteresis character of magnetic levitation system has been considered in mathematical modeling which is usually ignored in literature. Duhem model has been applied for modeling hysteresis of electromagnet. A useful derivative definition of Duhem model has been given for a smooth system which is necessary to apply feedback linearization method. Thus, the nonlinear and the linear controller have been designed for the system. Consequently, the hysteretic system tracked to the desired reference input function.

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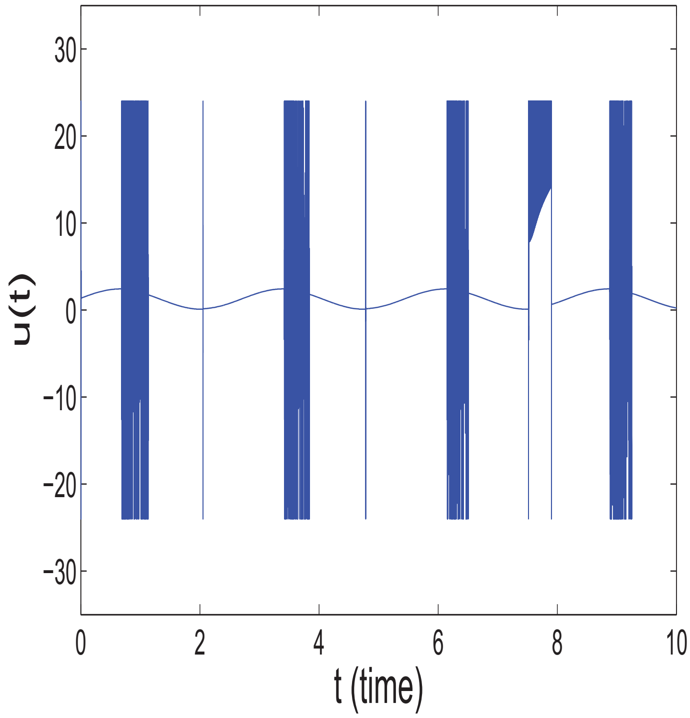

Figure 4: Output of the controller

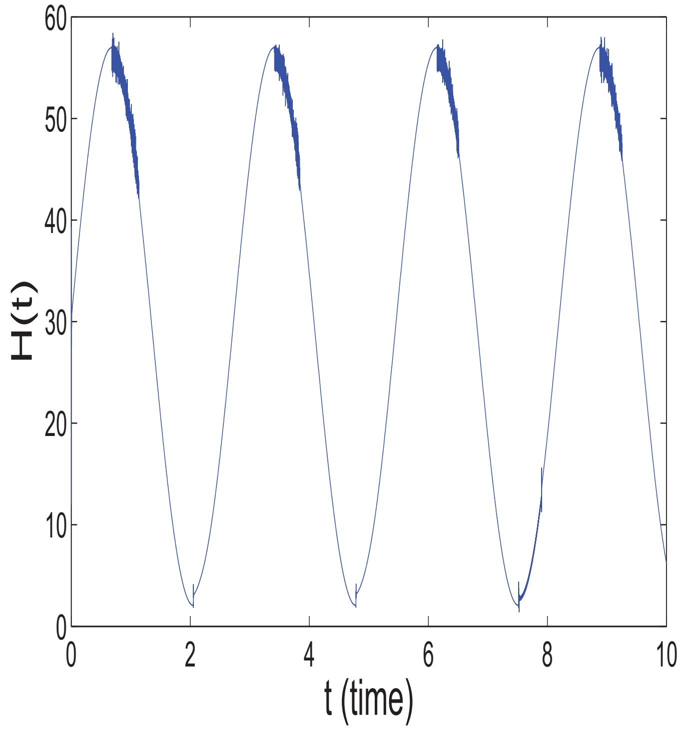

Figure 5: Produced magnetic field

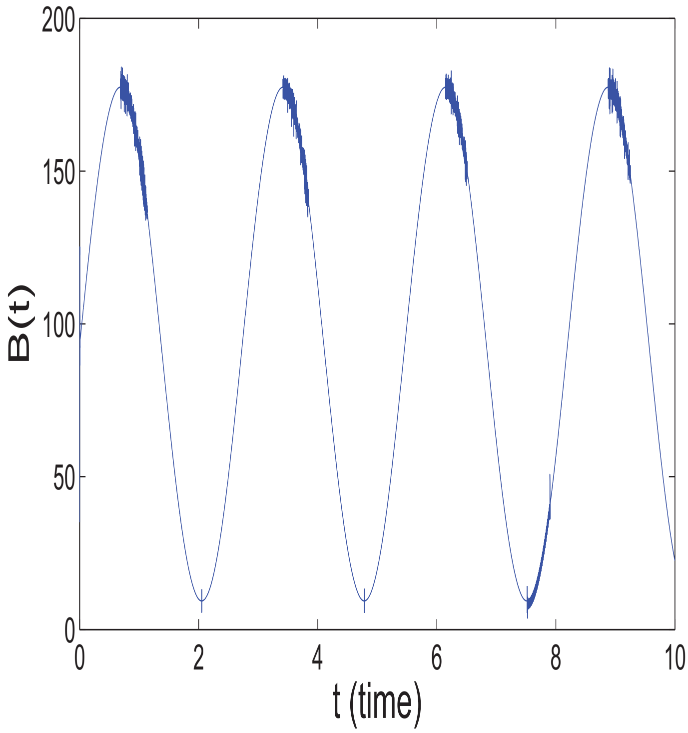

Figure 6: Produced magnetic induction

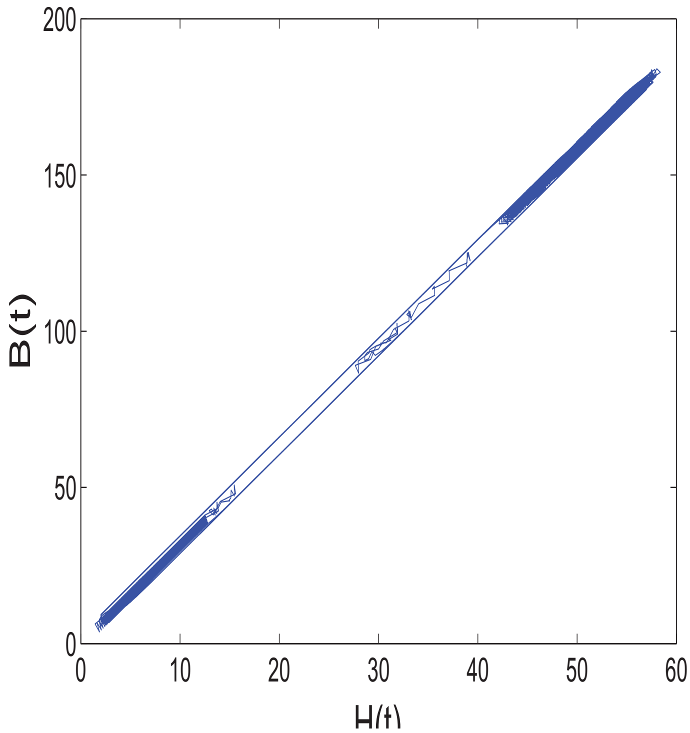

Figure 7: Hysteresis characteristic of controlled system

7 References

[1] Cho, D., Kato, Y., Spilmann, D.: *Sliding mode and classical control of magnetic levitation systems*, IEEE Control Systems, 13 (1993), 42-48.

[2] Mizuno, T., Takasaki, M., Kishita, D., Keiichiro, H.: *Vibration isolation system combining zero-power magnetic suspension with springs*, Control Engineering Practice, 15 (2007), 187-196.

[3] Golob, M., Tovornik, B.: *Modeling and control of the magnetic suspension systems*, ISA Transactions, 42 (2003), 89-100.

[4] Yang, J., Sun, R., Cui, J., Ding, X.: *Application of composite Fuzzy – PID algorithm to suspension systems of maglev train*, The 30th Annual Conference of the IEEE Industrial electronics Society, 2-6 November 2004, Busan, Korea.

[5] Charara, A., De Miras, J., Caron, B.: *Nonlinear control of a magnetic levitation system without premagnetization*, IEEE Transaction on Control Systems Technology, 4 (1996), 513-523.

[6] Kharaajoo, M.J., Tousi, M.M, Bagherzadeh, H., Ashari, A.E.: *Sliding mode control of voltage-controlled magnetic levitation systems*, IEEE, 0-7803-7729-X/03, (2003), 83-86.

[7] Li, T.H.S., Kuo, C.L., Guo, N.R.: *Design of an EP – based fuzzy sliding-mode control for a magnetic ball suspension system*, Chaos, Solutions and Fractals, 33 (2007), 1523-1531.

[8] Trumper, D.L., Olson, S.M., Subrahmanyan, P.K.: *Linearizing control of magnetic suspension systems*, IEEE Transactions on Control, Systems Technology, 5 (1997), 427-438.

[9] Joo, S., Seo, J.H.: *Design and Analysis of the nonlinear feedback linearizing control for an electromagnetic suspension system*, IEEE Transactions on Control Systems Technology, 5 (1997), 135-144.

[10] El Hajjaji, A., Ouladsine, M.: *Modeling and nonlinear control of magnetic levitation system*, IEEE Transactions on Industrial Electronics, 48 (2001), 831-838.

[11] Mittal, S., Menq, C.H.: *Hysteresis compensation in electromagnetic actuators through Preisach model inversion*, IEEE/ASME Transactions on Mechatronics, 5, (2000), 394-409.

[12] Zhou, J., Wen, C.: *Adaptive inverse control of a magnetic suspension system with input backlash*, 16th IEEE International Conference on Control Applications Part of IEEE Multi-Conference on Systems and Control, Singapore, 1-3 October 2007, 1347-1352.

[13] Colemann, B.D., Hodgdon, M.L.: *A constitutive relation for rate-independent hysteresis in ferromagnetically soft materials*, Internat. J. Engrg.Sci., 24 (1986), 897-919.

[14] Colemann, B.D., Hodgdon, M.L.: *On a class of constitutive relations for ferromagnetic hysteresis*, Arch. Rational Mech. Anal., 99 (1987), 375-396.

[15] Hodgdon, M.L: *Application of a theory of ferromagnetic hysteresis*, IEEE Trans. Magn., MAG-24, (1988), 218-221.

[16] Hodgdon, M.L: *Mathematical theory and calculations of magnetic hysteresis curves*. IEEE Trans. Magn., MAG-24, (1988), 3120-3122.

[17] Macki, J.W., Nistri, P., Zecca, P.: *Mathematical models of hysteresis*. SIAM Review, 35 (1993), 94-123.

[18] Visintin, A.: *Differential Models of Hysteresis*, (Eds.: John, F., Marsden J.E., Sirovich, L.), Springer, Berlin, 1994, 130-150.

[19] Marino, R., Tomei, P.: *Nonlinear Control Design Geometric*, Adaptive and Robust: Prentice Hall, 1995.