# IMPROVED ROLL TO ROLL MODELLING FOR ELASTIC WEBS

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**Abstract**. The systems handling web material such as fabric, paper, polymer or metal sheets are common in the manufacturing industry. Modelling and control of roll to roll systems have been studied for several decades. However, increasing requirements on control performances and better handling of elastic web material have led to search for more sophisticated control strategies and therefore for more precise roll to roll modelling. In the published models, the elastic web strain between two consecutive rolls is considered as positive. Nevertheless, the case of negative web strain can appear in transient phases when low web tension is required. A new dynamic web model is presented in this paper which includes the influences of web slacks. This new model is useful in simulator constructions and modern control strategies synthesis.

## **1** Introduction

The systems transporting thin and flexible materials such as fabric, paper, polymer or metal sheets are very common in the industry. The key points in the web handling industry are to increase the web transport velocity as much as possible while controlling the tension of the web. The main concern is the coupling existing between web velocity and tension.

Modelling and control of web handling systems have been studied for several decades [9]. However, increasing requirement on control performance and better handling of elastic web material have led to search for more so-phisticated robust control strategies [9][5] and therefore for more precise roll to roll modelling [9]. Especially control strategies based on phenomenological models (for example feedforward control) require precise plant modelling [5][7].

The nonlinear model of a web transport system is built from the classical equations describing the web tension behaviour between two consecutive rolls and the velocity of each roll. The web elasticity is described by the Hooke's law. The viscosity effects can also be included (for example the Maxwell or Voigt models). The web is assumed perfectly flexible with no bending stiffness. Moreover, one major hypothesis is that there is no sliding between web and rolls. The established model was identified on an experimental bench [1].

In the published models, the elastic web strain between two consecutive rolls (and therefore the web tension) is considered as positive. Nevertheless, the case of negative web strain can appear in transient phases when low web tension is required. A new dynamic web model is presented in this paper which includes the influences of web slacks.

#### 2 Classical roll to roll modelling

The nonlinear model [1] of a web transport system is built from the equations describing the web tension behaviour between two consecutive rolls and the velocity of each roll. This model was identified on an experimental bench [1].

**Web tension calculation :** The dynamics of web tension between two rolls of web transport systems is based on three laws [9]:

• *Hooke's law*: the tension T of an elastic web is a function of the web strain  $\varepsilon$ , the Young modulus E of elasticity, the web cross section S and the web length L between two consecutive rolls :

$$T = ES\varepsilon = ES\frac{L - L_0}{L_0} \tag{1}$$

L and  $L_0$  are stretched and unstretched web lengths, respectively

- *Coulomb's law*: the study of the web tension on a roll can be considered as a problem of friction between solids [4].
- Equation of Continuity applied to the web between two driven rolls, yields [1][5]:

$$L \frac{d}{dt} \left( \frac{1}{1 + \varepsilon_k} \right) = \frac{V_{k-1}}{1 + \varepsilon_{k-1}} - \frac{V_k}{1 + \varepsilon_k}$$
(2)

where *k* is the span number

The relation (2) can be programmed in the Matlab software environment, as illustrated in figure 1 :



Figure 1. Web strain calculation (in Matlab environment) for a given span.

Web velocity calculation : The linear velocity  $V_k$  of roll k is obtained from the torque balance [1] :

$$\frac{d(J_k \frac{V_k}{R_k})}{dt} = R_k (T_{k+1} - T_k) + K_k U_k + C_f$$
(3)

where  $K_k U_k$  is the motor torque and  $C_f$  is the friction torque. In the case of unwind and rewind rolls, the inertia and radius are time dependent and can substantially vary during processing.

A large scale web handling system of any number of driven rolls can be built from the equations (1) to (3).

**Identification** : The identification of the complete nonlinear parametric model is based on the model matching approach. It is done in several steps in order to reduce the number of parameters to identify at each step and, consequently, simplify the optimization of the parameters [1].



Figure 2. Experimental setup with 3 motors and 2 load cells.



Figure 3. New web strain calculation (in Matlab environment) for a given span.

## 3 New roll to roll modelling

The strain dynamic of the web can be described with equation (2), where L is the distance between two consecutive rolls. For flexible webs, negative web strain can not appear in a real plant and consequently this equation is

useful only for positive strain (and positive web tension). Classically, the resolution of equation (2) includes a saturation that eliminates negative strains. The drawback is that the web slack is not taken into account. A new approach including the web slack calculation (as indicated in equation (4)) and the effects of the slack has been established. The modifications of equation (2) are represented on Figure 4.

$$\Delta L_{k} = L + \int_{t_{0}}^{t} (V_{k} - V_{k+1}) dt$$
(4)



Figure 4. New web strain calculation (in Matlab environment) for a given span (the modifications are represented in red)



Figure 5. Web tension calculation : comparison between the classical strain equation (green) and the new model (blue) given in Figure 2.

Figure 5 represents the winding tension simulated with a 3-motor plant simulator. In our case, web slacks appear and they have to be eliminated with the decentralized controllers. The new approach gives the precise web dynamic behaviour whereas the classical web equation leads to negative tensions. The simple elimination, with saturation, of these negative tensions in the model is not realistic, as seen in Figure 5.

#### 4 Conclusion

An improved roll to roll modelling for elastic webs is presented in this paper. It reproduces the web dynamics (tension and velocity) even in the case of web slacks. This approach is useful in realistic simulator constructions and modern control strategies based on phenomenological models.

## **5** References

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