MODELING OF NONLINEAR INDUCTANCE IN DYNAMIC TRANSIENT SIMULATIONS

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Abstract. Ordinary ways for modeling of nonlinear electrical elements in electrical network transient calculations are: using known analytical function to approximate of nonlinearity or using piecewise linear representation of nonlinear curve. Improved simulation speed and numerical stability properties is reason for using piece-wise linearizing of nonlinear electrical elements in this paper. An equivalent model for representing the nonlinear inductance using a linear inductance and a corresponding current source is developed in this paper. The model is functionally dependent on the position of the operating point *k*. At the same time, the inductance model is not dependent on the integration step, which is opposite to EMTP-like models. Differential equation systems, which describe the electromagnetic transients in transformers, written in a state space form, are usually stiff or very stiff. BDF numerical methods are preferred for simulations of these stiff systems. Developed model of nonlinear inductance, implemented into single-phase transformer model, has shown good simulation results compared to measured results. The developed model can be further generalized for the other single- or three-phase nonlinear electrical elements. Furthermore, suggested model may be implemented in some of commercial software for dynamical analyses of electrical systems such as MATLAB/SPS or EMTP-ATP.

1 Introduction

The modeling of nonlinear electrical elements in electrical network transient calculations can be done by using known analytical function for approximation of nonlinearity or by using piecewise linear representation of nonlinear curve. Therefore, exponential function for representing nonlinear inductance model is used in the paper [1], in [2] for modeled surge arrester. The piecewise linearization of nonlinear curve for modeling of electrical elements, on the other hand, is used in the papers [3],[4]. Linearizing of nonlinear curve compared to using nonlinear analytical function has advantage and disadvantage. A successful control of numerical stability properties of applied numerical methods is the main advantage [5]. Also, another advantage of this linearizing, is improved speed compared to classical methods used in calculations within nonlinear algebraic-differential equation systems. On the other hand, piece-wise linearization has basic disadvantage related to negative overshooting effects or jumping of certain linear segments [5].

2 Piece-wise modelling of nonlinear electrical elements

The piecewise linearizing of nonlinear electrical elements is preferred in this paper. In fact, this way of modeling in comparison to standard modeling with nonlinear analytical functions, is set on superior position by the mentioned advantages of this kind of nonlinear elements modeling. Assume that nonlinear electrical element is presented by measured sample sets (x_i, y_i), $i = 1,2,...,N$.

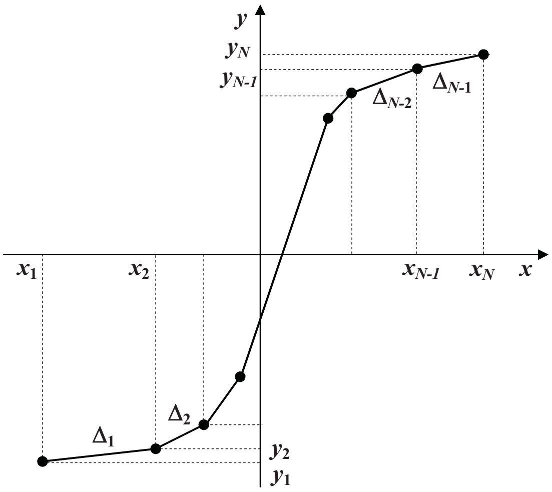


Figure 1. Piece-wise linearized nonlinear curve

In this case, the following general relation on the *k*-th linear region is valid:

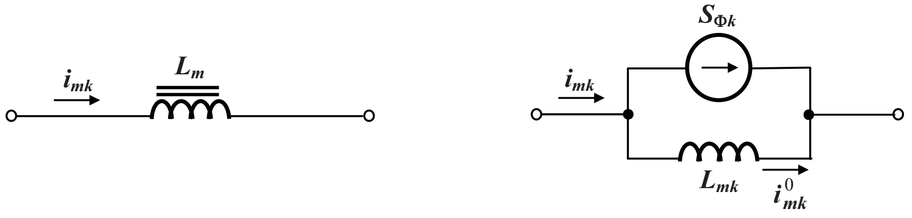
$$x = \frac{1}{\Delta_k} y + X_k \tag{1}$$

where are: $x_k \le x \le x_{k+1}$, $y_k \le y \le y_{k+1}$, $\Delta_k = \dfrac{y_{k+1} - y_k}{x_{k+1} - x_k}$, $X_k = x_k - \dfrac{1}{\Delta_k} y_k$.

Based on this simple relation, it is possible to create an equivalent model for both nonlinear resistance and nonlinear inductance [5] (with or without hysteresis effects [6]). For example, the following relation is valid for nonlinear inductance, by single valued odd-symmetry sample sets, in the first quadrant, Figure 1:

$$i_{m_k} = \frac{1}{L_{m_k}} \Phi + \operatorname{sgn}(\Phi) \cdot I_{m_k} = \frac{1}{L_{m_k}} \Phi + S_{\Phi_k} \tag{2}$$

It is possible to create an equivalent model for representing the nonlinear inductance using a linear inductance and a corresponding current source, based on this model, Figure 2. The model is functionally dependent on the position of the operating point *k*. At the same time, the inductance model is not dependent on the integration step, which is opposite to EMTP-like models [7].



(a) (b)

Figure 2. (a) Non-linear inductance, (b) equivalent model

The proposed model of nonlinear inductance is relatively easy to implement in state space form. Furthermore, differential equation systems, which are used to describe the behaviour of electromagnetic transients, written in a state space form, are usually stiff or very stiff. By definition, stiff systems are systems for which the ratio of extreme eigenvalues of the system matrix is $\left|\mathrm{Re}(\lambda_i)\right|_{\max} / \left|\mathrm{Re}(\lambda_i)\right|_{\min} >> 1$, while very stiff systems have at least one very big negative eigenvalue, e.g. $\left|\mathrm{Re}(\lambda_i)\right|_{\max} >> 1$. A typical stiff or very stiff system of differential equations can be found in calculations of low frequency transformer transients [5]. Stiff systems require being solved using *A*-stable numerical methods, to ensure numerical stability. However, it is necessary that very stiff systems need to be solved by *L*-stable numerical methods. The *A*-stable methods applied to very stiff systems have drawbacks because of generation of numerical oscillations. The trapezoidal rule, as a traditional numerical method used in EMTP-like programs, is not a *L*-stable method, but in fact an *A*-stable method. But, on the other hand, Backward Differential Formulas (BDF) represents an *A*-, as well as a *L*-stable method:

$$\sum_{m=1}^{p} \frac{1}{m} \nabla^m x_{k+1} = \Delta t \, f_{k+1} \tag{3}$$

3 Model validation

Developed model of nonlinear inductance is tested on the simple example of single-phase transformer energization, Figure 3. Following the parameters of model.

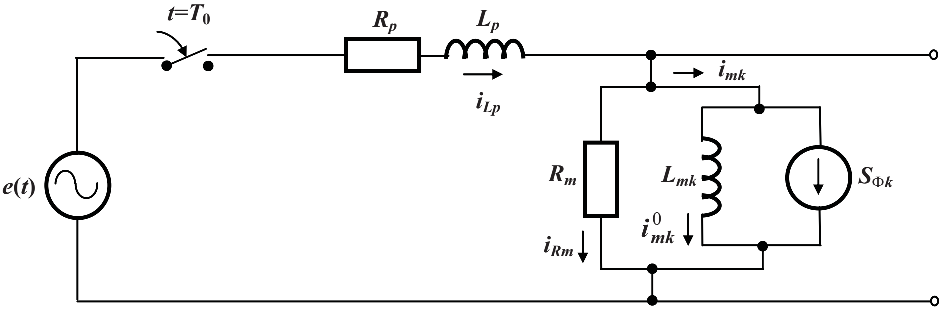


Figure 3. Model of single-phase transformer

The transformer parameters:

- Nominal transformer power: $S_{tr} = 300\ VA$;
- Primary voltage: $U_p = 220\ V$;
- Secondary voltage: $U_s = 24\ V$;
- Short circuit voltage $u_{k\%} = 5.45\ \%$;
- Active resistance of primary winding (includes the resistance of input cable) $R_p = 3.75\ \Omega$;
- Leakage inductance of primary winding (obtained from the short circuit test) $L_p = 2.54\ mH$;
- Active resistance that is equivalent to the total losses (i.e. hysteresis losses and eddy current losses) in the transformer iron core $R_m = 3826\ \Omega$;

Magnetizing curve of transformer iron core is represented in Figure 4.

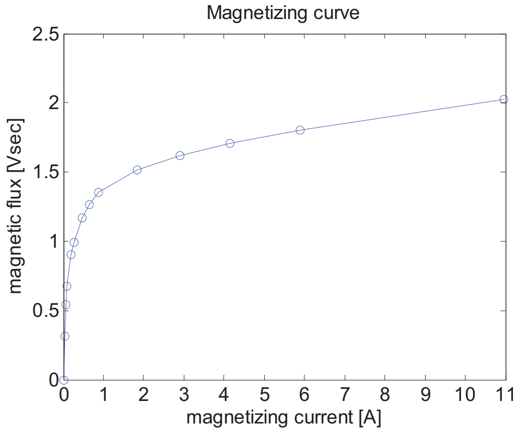


Figure 4. Magnetizing curve of transformer iron core

Network voltage signal is an equivalent to sinusoidal function of the following form:

$$e(t) = 328\cos(\omega t - 39^\circ)$$

State space form for dynamical behaviour of transformer is:

$$\frac{d}{dt}\begin{bmatrix} i_p(t) \\ \Phi(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{R_p + R_m}{L_p} & \dfrac{R_m}{L_p L_{m_k}} \\ R_m & -\dfrac{R_m}{L_m L_{m_k}} \end{bmatrix} \begin{bmatrix} i_p(t) \\ \Phi(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_p} & \dfrac{R_m}{L_p} \\ 0 & -R_m \end{bmatrix} \begin{bmatrix} e(t) \\ S_{\Phi_k} \end{bmatrix} \tag{4}$$

Figure 5 shows eigenvalues distribution per the linear region of magnetizing curve. Since it is obvious that we are dealing with very stiff system, the equation (4) is solved with BDF2 method of second order.

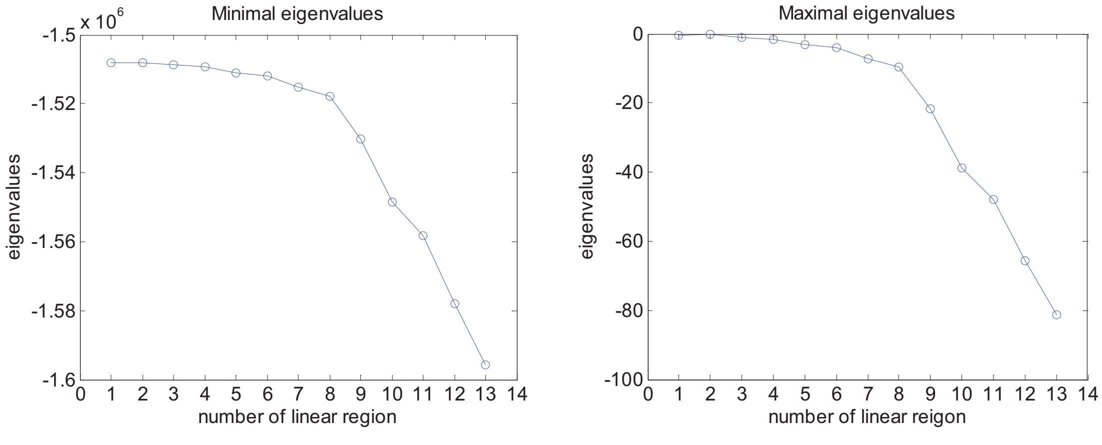
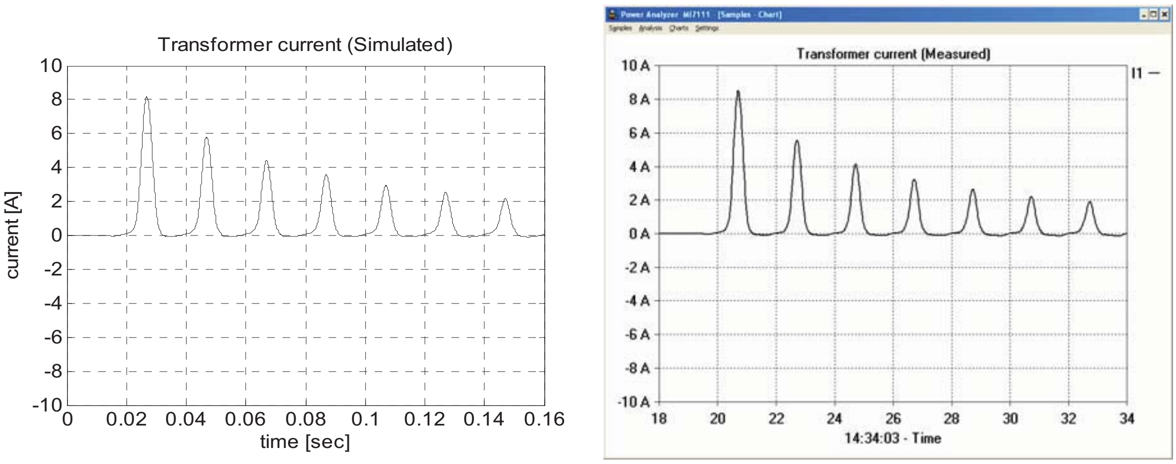


Figure 5. Eigenvalues distribution per the linear region of magnetizing curve

The next Figure 6 shows measured and simulated results obtained by developed model of nonlinear inductance. The inrush current waveform of unloaded transformer has been analysed.



(a) (b)

Figure 6. Transformer inrush current: (a) measured and (b) simulated

4 Conclusions

Developed model of nonlinear inductance, implemented into single-phase transformer model, has shown good simulation results compared to measured results of transformer inrush current. The developed nonlinear inductance model can be further generalized for the other nonlinear electrical elements such as surge arresters, diodes, transistors, thyristors, etc. Further, the suggested model can be relatively easy applied on the three-phase electrical systems. At the end, because of its flexibility and simplicity, the suggested model is possible to implement in some of commercial software for dynamical state analyses of electrical systems, such as MATLAB/SymPowerSystem or EMTP-ATP.

5 References

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