FREE VIBRATION OF 3-D PIERCED SHEAR WALLS WITH CHANGES IN CROSS-SECTION USING CONTINUOUS CONNECTION METHOD

O. Aksogan[1], C.D. Turkozer[1], E. Emsen[2]
[1]Departmant of Civil Engineering, University of Cukurova, 01330 Adana, Turkey
[2]Departmant of Civil Engineering, Akdeniz University, 07058 Antalya, Turkey

Corresponding Author: O. Aksogan, Departmant of Civil Engineering, University of Cukurova, 01330 Adana, Turkey; oaksogan@cu.edu.tr

Abstract. The paper considers free vibration analysis of non-planar coupled shear walls resting on rigid foundations. The analysis considers coupled shear walls with changes in cross-section, the properties of which vary from region to region along the height. In this study, continuous connection method (CCM) and Vlasov's theory of thin-walled beams are employed to find the structure stiffness matrix. The structure mass matrix is found with the lumped mass idealization. While the discrete structure is formulated as a continuous medium, the continuously distributed mass of the structure is discretized to a system of lumped masses for finding the corresponding stiffness matrix. After obtaining the standard frequency equation of the discrete system, the circular frequencies are determined in a straightforward manner and used to find the modes of vibration. A computer program has been prepared in Fortran Language to implement the foregoing analysis. The structure is solved both by the present method using CCM and by the SAP2000 structural analysis program using the frame method. It is observed that the results obtained by the present method coincide with those of SAP2000 structural analysis program perfectly. This method is an effective method in terms of simplicity of its data and extremely short computation time for the predesign of high-rise buildings.

1 Introduction

Shear walls are used to resist the lateral loads that arise from the effect of winds and earthquakes. Hence, shear walls have become very popular in high-rise buildings. However, the design of shear walls weakened by doors, windows and corridor openings is generally unavoidable in structural engineering. These features turn a simple shear wall into a coupled pair, which can be considered as two smaller walls, coupled together by a system of lintel beams.

When the coupling action between the piers separated by openings becomes important, some of the external effects are resisted by the internal forces and moments in the walls due to the increase in the stiffness of the coupled system by the connecting beams. Actually, the deformation of a pierced shear wall subjected to lateral loading is not confined to its plane. Studies considering in-plane, out-of-plane and torsional deformation in the investigation of pierced shear walls are called 3-D shear wall analyses. In 3-D pierced shear walls, both the flexural and torsional behaviours under external loading have to be taken into account in the anaysis.

All of the dynamic analysis in the literature on perforated shear walls concern themselves with planar ones [1]. No study has been made, to the knowledge of the authors, concerning the dynamic anaysis of 3-D pierced shear walls, so far.

The present analyses is based on the Continuous Connection Method (CCM), in conjunction with Vlasov's theory of thin-walled beams, following an approach similar to the one used by Tso and Biswas [2]. In the CCM, the connecting beams are assumed to have the same properties and spacing along the entire height of the wall. The discrete system of connecting beams is replaced by continuous laminae of equivalent stiffness [3]. The CCM has been employed in the analysis and the compatibility equation has been written at the mid-point of the connecting beams. For this purpose, the connecting beams have been replaced by an equivalent layered medium. The warping of the piers due to their twist, as well as their bending, has been considered in obtaining the displacements. Vlasov's thin walled beam has been used for this purpose [4].

2 Free Vibration Analysis

Free vibration analysis of non-planar coupled shear walls in conjunction with the CCM consists of two steps. In the first step, the structure is considered as a discrete system of lumped masses at the selected levels along the height of the structure (see Figure 1).

Lumped masses are concentrated at the center of the whole cross-sectional area of the structure. Since each point has three degrees of freedom, in X, Y and Teta directions, the dimension of mass matrix is equal to 3nx3n, where n represents the number of masses. However, the mass matrix elements associated with the rotational degrees of freedom will be zero because of the assumption that the mass is lumped at nodes which have no rotational inertia. Thus, the lumped-mass matrix is a diagonal matrix which has zero diagonal elements for the rotational degrees of freedom.

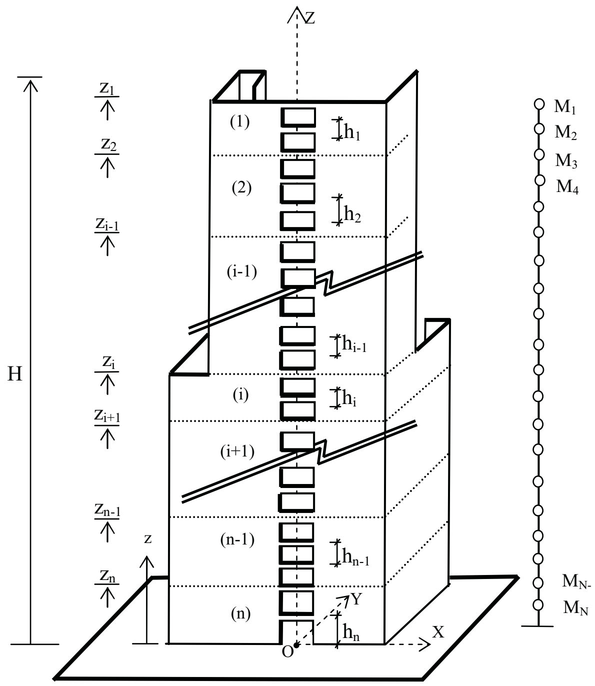

Figure 1. Non-planar coupled shear wall and its lumped mass model

The mass matrix of the coupled shear wall was found as a diagonal matrix employing a lumped mass approach. To explain this procedure, the top, bottom and each height at which there is a stiffening beam and/or change of wall thickness will be called ‘‘ends’’ and the section between any two consecutive ends will be called a ‘‘region’’. Each region is divided into suitable numbers of parts with corresponding amounts of masses.

After the determination of the mass matrix, the second step is the determination of the stiffness matrix of the structure for the degrees of freedom chosen during the determination of the mass matrix. This procedure is carried out by applying two horizontal unit forces in the directions of X and Y axes and one unit moment about Z axis at every height with a lumped mass. For every one of these loadings, a solution is carried out making use of the CCM and writing down the compatibility equation for the vertical displacements at the midpoints of the connecting beams. Then, employing the equilibrium equations, the corresponding displacements are obtained. The displacements of the points where the lumped masses are located are determined by using the rigid floor diaphragm assumption. Thus, each unit loading gives one column of the flexibility matrix as the displacements at the points where the lumped masses are. Hence, the analysis for the three loading cases for one floor will suffice to introduce the complete solution procedure for the flexibility matrix. The stiffness matrix of the structure will be determined by taking the inverse of the flexibility matrix. Substituting the mass and stiffness matrices, thus obtained, in the equations of motion for free vibration, the system of equations for the problem in hand is obtained.

The basic assumptions of the CCM for non-planar coupled shear walls can be summarized as follows:

* The geometric and material properties are constant throughout each region i along the height.
* The discrete set of connecting beams with bending stiffness EIci in region i are replaced by an equivalent continuous connecting medium of flexural rigidity EIci/hi per unit length in the vertical direction.
* Vlasov’s theory for thin-walled beams of open section is valid for each pier.
* The outline of a transverse section of the coupled shear wall at a floor level remains unchanged in plan (due to the rigid diaphragm assumption for floors).
* The discrete shear forces in the connecting beams in region i are replaced by an equivalent continuous shear flow function q_i, per unit length in the vertical direction along the mid-points of the connecting laminae.

* The torsional stiffness of the connecting beams is neglected.
* The walls and beams are assumed to be linearly elastic.
* Bernoulli-Navier hypothesis is assumed to be valid for the connecting beams.

The axial force in each pier is found by writing down the vertical force equilibrium equation for the part of one pier above any horizontal cross-section as

$$T_i = \int_z^{z_i} q_i dz + \sum_{t=1}^{i-1} \left(\int_{z_{t+1}}^{z_t} q_t dz \right) \qquad (i = 1,2,...,n) \tag{1}$$

A cut through the points of contra-flexure of laminae exposes the shear flow q_i. The vertical force equilibrium of a dz element of one pier yields the relation

$$q_i = -T_i' \qquad (i = 1,2,...,n) \tag{2}$$

where a prime denotes differentiation with respect to z.

2.1 Compatibility Equation

While obtaining the compatibility equations, all connecting laminae are cut through their mid-points, O', which are the points of zero moment.

The vertical displacement due to bending can be obtained as the product of the slope at the section considered and the distance of point O' from the respective neutral axis. In addition, vertical displacement arises, also, due to the twisting of the piers, and is equal to the value of the twist at the section considered, times the sectorial area, ω, at point O'.

For the compatibility of displacements, the relative vertical displacements of the cut ends must be equal to zero. Hence,

$$u_i'a + v_i'b + \theta_i'(\omega + d) - \frac{1}{E}\sum_{j=i+1}^{n}\left[\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_{z_{j+1}}^{z_j} T_j\, dz\right] - \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\int_{z_{i+1}}^{z} T_i\, dz + \frac{T_i'}{E}\left[\frac{h_i c^3}{12 EI_{c_i}} + \frac{1.2 h_i c}{GA_{c_i}}\right] = 0 \tag{3}$$

in which

$$\omega = \omega_1 - \omega_2 \tag{4}$$

$$a = x_{g_2} - x_{g_1}, \qquad b = y_{g_2} - y_{g_1}, \qquad d = x_{s_2} y_{g_2} - y_{s_2} x_{g_2} + y_{s_1} x_{g_1} - x_{s_1} y_{g_1} \tag{5}$$

and ω_1 and ω_2 are the sectorial areas at points on the left and right sides of the cut for piers 1 and 2, respectively. u_i, v_i and θ_i are the global displacements ($i = 1,2,...,n$). Differentiating this equation with respect to z and letting

$$\gamma_i = \frac{h_i c^3}{12\, EI_{c_i}} + \frac{1.2\, h_i c}{GA_{c_i}} \tag{6}$$

the following equation is obtained:

$$u_i''a + v_i''b + \theta_i''(\omega + d) - \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)T_i + \frac{T_i''}{E}\gamma_i = 0 \qquad (i = 1,2,...,n) \tag{7}$$

The successive terms in (3) represent the contributions of the bending of the piers about the principal axes, the contribution of the twisting of the piers, the axial deformation of the piers, the bending deformation in the laminae and the shearing deformation in the piers, respectively.

2.2. Equilibrium Equations

The coordinate system and the positive directions of the internal bending moments acting on the different components of the shear wall are adapted as shown vectorially in Figure 2.

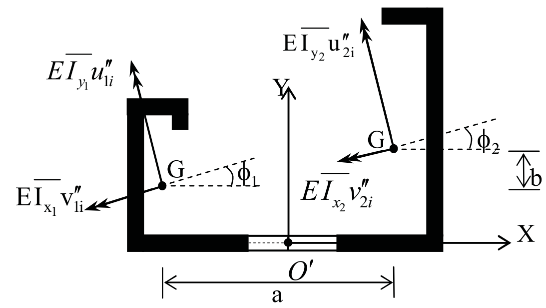

Figure 2. Internal bending moments

These internal moments, along with the couple produced by the axial force, T_i, balance the external bending moments M_{EX_i} and M_{EY_i}. For the equilibrium of the moments about the X and Y axes can be written as,

$$M_{EY_i} = E\,I_Y u''_i + E\,I_{XY} v''_i - E\,I_{\theta Y}\theta''_i + T_i\, a \quad , \tag{8}$$

$$M_{EX_i} = E\,I_{XY} u''_i + E\,I_X v''_i + E\,I_{\theta X}\theta''_i + T_i\, b \quad , \tag{9}$$

where

$$I_Y = I_{y_1} + I_{y_2} \quad , \quad I_X = I_{x_1} + I_{x_2} \quad , \quad I_{XY} = I_{xy_1} + I_{xy_2} \tag{10}$$

$$I_{\theta X} = x_{s_1} I_{x_1} + x_{s_2} I_{x_2} - y_{s_1} I_{xy_1} - y_{s_2} I_{xy_2} \; , \; I_{\theta Y} = y_{s_1} I_{y_1} + y_{s_2} I_{y_2} - x_{s_1} I_{xy_1} - x_{s_2} I_{xy_2} \tag{11}$$

In these equations, I_{y_j} and I_{x_j} are the second moments of area of the cross-sections, and I_{xy_j} is the product of inertia of pier j (j=1, 2) about axes parallel to the global axes and passing through the centroids.

In order to obtain the bimoment equilibrium equation, the coupled shear wall will be cut through by a horizontal plane such that an upper part is isolated from the lower part of the structure. Equating the external bimoment, B_{E_i}, to the internal resisting bimoments, the bimoment equilibrium equations for all regions of the structure can be written as follows:

$$B_{E_i} = \overline{B}_i + \overline{\overline{B}}_i \quad , \quad B_{E_i} = E\,I_{\theta Y} u''_i - E\,I_{\theta X} v''_i - E\,I_\omega \theta''_i - (\omega + d)T_i \qquad (i = 1,2,...,n) \tag{12}$$

where, $\overline{B}_i$ is the resultant bimoment about point O', which is due to the resistance offered by the piers and $\overline{\overline{B}}_i$ is the resultant bimoment due to the additional bending moments and bimoments about the vertical axis through point O'. I_ω is the sectorial moments of inertia of the two piers.

In order to obtain the twisting moment equilibrium equation, the coupled shear wall will be cut through by a horizontal plane such that an upper free body diagram is isolated from the rest of the structure. Equating the external twisting moment, M_{Et_i}, to the internal resisting moments, the twisting moment equilibrium equation for all regions of the structure can be written as follows:

$$M_{Et_i} = \overline{M_{t_i}} + \overline{\overline{M_{t_i}}} \tag{13}$$

$$M_{Et_i} = E\,I_{\theta Y_i} u'''_i - E\,I_{\theta X_i} v'''_i + G J_i \theta'_i - E\,I_{\omega_i}\theta'''_i - (\omega_i + d_i)T'_i \quad (i = 1,2,...,n) \tag{14}$$

where, $\overline{M_{t_i}}$ is the resultant torque about the vertical axis through point O', which is due to the resistance offered by the piers and $\overline{\overline{M_{t_i}}}$ is the resultant twisting moment due to the additional torques about point O' and the shear forces. Expressions $GJ_i\theta_i'$ (i=1,2,…,n) are the St. Venant twisting moments and expressions $-E\overline{I_{\omega_i}}\theta_i'''$ (i=1,2,…,n) are the additional twisting moments due to the non-uniform warping of the piers along the height.

Using the compatibility equation (3) and the four equilibrium equations (8), (9), (12), and (14), the 4n unknowns of the problem, namely u_i, v_i, θ_i, and T_i, can be found under the applied loadings M_{EX_i}, M_{EY_i}, B_{E_i}, and M_{Et_i}. The elimination of u_i, v_i and θ_i from equations (7,8,9,12,14) yields the following differential equation for T_i:

$$(\beta_{1i})T_i'''' - (\beta_{2i})T_i'' + (\beta_{3i})T_i = -M_{EY_i}''\left(\overline{I}_\omega K_3 + K_1 r\right) - M_{EX_i}''\left(\overline{I}_\omega K_4 - K_2 r\right) + \frac{GJ}{E}\left(M_{EY_i}K_3 + M_{EX_i}K_4\right) + M_{Et_i}' r \tag{15}$$

in which

$$\beta_{1i} = \gamma_i \overline{I}_\omega \quad , \quad \beta_{2i} = \frac{\overline{I}_\omega}{A} + \frac{GJ\gamma_i}{E} + r^2 \quad , \quad \beta_{3i} = \frac{GJ}{EA} \tag{16}$$

$$K_1 = \frac{(I_X I_{\theta Y} + I_{XY} I_{\theta X})}{\Delta} \quad , \quad K_2 = \frac{(I_{XY} I_{\theta Y} + I_Y I_{\theta X})}{\Delta} \quad , \quad K_3 = \frac{(aI_X - bI_{XY})}{\Delta} \quad , \quad K_4 = -\frac{(aI_{XY} - bI_Y)}{\Delta}$$

$$r = \omega + d + aK_1 - bK_2 \quad , \quad \overline{I}_\omega = I_\omega - I_{\theta X}K_2 - I_{\theta Y}K_1$$
$$\frac{1}{A} = \left[\frac{1}{A_1} + \frac{1}{A_2}\right] + aK_3 + bK_4 \quad , \quad \Delta = \left(I_X I_Y - I_{XY}^2\right) \tag{17}$$

Thus, the governing differential equation of the analysis of non-planar stiffened coupled shear walls is found as equation (15). When this equation is used for a unit loading, $\mathrm{M}_{\mathrm{EX_i}}$ and $\mathrm{M}_{\mathrm{EY_i}}$ are the external bending moments and $\mathrm{M}_{\mathrm{Et_i}}$ is the external twisting moment about the respective global axes for the particular unit loading. Equation (15) is written for each region separately. However, in this context when the unit load is applied at an internal point of a region, it divides that region into two new regions. The system of Macaulay's brackets should be understood, here and in the sequel, as

$$\langle z - z'\rangle^n = (z - z')^n \quad \text{and} \quad \langle z - z'\rangle^0 = 1 \quad \text{for} \quad z > z'$$
$$\langle z - z'\rangle^n = 0 \quad \text{and} \quad \langle z - z'\rangle^0 = 0 \quad \text{for} \quad z \le z' \tag{18}$$

Thus, in the general form, the external effects $\mathrm{M}_{\mathrm{EX_i}}$, $\mathrm{M}_{\mathrm{EY_i}}$ and $\mathrm{M}_{\mathrm{Et_i}}$ for any unit loading is found, using the following expressions for the particular case:

$$M_{EX_i} = \langle H_p - z\rangle^1$$
$$M_{EY_i} = \langle H_p - z\rangle^1 \tag{19}$$
$$M_{Et_i} = \langle\ (-d_{PY}) + (d_{PX})\ \rangle^1$$

where, d_{PX} and d_{PY} are the moment arms of the components of the unit force from point O'.

Employing the Macaulay's brackets,

$$\text{if} \quad H_p > z \quad ; \quad \langle H_p - z\rangle^1 = (H_p - z)$$
$$\text{if} \quad H_p \le z \quad ; \quad \langle H_p - z\rangle^1 = 0 \tag{20}$$

in accordance with the system of Macaulay's brackets, we can rewrite M_{EX_i} and M_{EY_i} for the part beneath the unit load as follows:

$$
\begin{aligned}
M_{EX_i} &= (H_p - z) \qquad M'_{EX_i} = -1 \qquad M''_{EX_i} = 0 \\
M_{EY_i} &= (H_p - z) \qquad M'_{EY_i} = -1 \qquad M''_{EY_i} = 0
\end{aligned}
\tag{21}
$$

Hence, for the part above the unit load, M_{EX_i}, M_{EY_i} and M_{Et_i} are equal to zero.

Substituting expressions (19) in (15) and solving the resulting differential equation, T_i is found as follows:

$$
T_i = D_{1i}\, Sinh[\alpha_{1i}\, z] + D_{2i}\, Cosh[\alpha_{1i}\, z] + D_{3i}\, Sinh[\alpha_{2i}\, z] + D_{4i}\, Cosh[\alpha_{2i}\, z] + \frac{1}{\beta_{3i} E}\left[GJ_i\left(K_{3i} M_{EY_i} + K_{4i} M_{EX_i}\right)\right]
\tag{22}
$$

in which

$$
\alpha_{1i} = \sqrt{\left(\frac{\beta_{2i} - \sqrt{\beta_{2i}{}^2 - 4\beta_{1i}\beta_{3i}}}{2\beta_{1i}}\right)} \quad , \quad \alpha_{2i} = \sqrt{\left(\frac{\beta_{2i} + \sqrt{\beta_{2i}{}^2 - 4\beta_{1i}\beta_{3i}}}{2\beta_{1i}}\right)}
\tag{23}
$$

To determine the integration constants D_{1i} to D_{4i} in the single fourth order differential equation, the boundary conditions at the top, bottom and between each pair of consecutive regions are employed. Substituting them in expression (22), the general solution for T_i $(i = 1, 2, \ldots, n)$ can be found using (8,9,14) as follows:

$$
\theta_i = \frac{1}{Er_i}\int\left(\int\left[-M_{EX_i}K_{3i} - M_{EX_i}K_{4i} + \frac{T_i}{A_i} - T''_i\gamma_{c_i}\right]dz\right)dz + G_{1i}z_i + G_{2i}
\tag{24}
$$

$$
u_i = \frac{1}{E}\int\left(\int\left[E\,\theta''_i K_{1i} - T_i\,K_{3i} - \frac{M_{EX_i}\,I_{XY_i}}{\Delta_i} + \frac{M_{EY_i}\,I_{X_i}}{\Delta_i}\right]dz\right)dz + R_{1i}z_i + R_{2i}
\tag{25}
$$

$$
v_i = \frac{1}{E}\int\left(\int\left[-E\,\theta''_i K_{2i} - T_i K_{4i} - \frac{M_{EY_i}\,I_{XY_i}}{\Delta_i} + \frac{M_{EX_i}\,I_{Y_i}}{\Delta_i}\right]dz\right)dz + N_{1i}z_i + N_{2i}
\tag{27}
$$

where boundary conditions and the equivalence of the horizontal displacements and the respective slopes for every pair of neighbouring regions at their common boundary ($z = z_i$) are used to determine the integration constants.

Having determined the displacements for unit loadings at each and every one of the levels of lumped masses, the flexibility matrix, and thereby, the stiffness matrix of the structure can be found. Finally, the circular frequencies are determined from the following standard frequency equation for the lumped mass system:

$$
\left|\underline{K} - \omega^2 \underline{M}\right| = 0
\tag{28}
$$

where ω is the circular frequency, M is the mass matrix and K is the stiffness matrix of the structure. The respective modal vectors, s_i, are found by substituting each and every circular frequency, ω_i, in the following equation at a time:

$$
\left(\underline{K} - \omega_i{}^2 \underline{M}\right)s_i = 0 \qquad i = 1,2,\ldots,m
\tag{29}
$$

3 Numerical application

The example problem compares the free vibration analyses of 3-D pierced shear walls with changes in cross-section using the present program and the SAP2000 [5] structural analysis program.

In this example, the stories above the fourth are of a different cross-section than the ones below as shown in Figure 3.

The total height of the shear wall is 24 m, the storey height is 3 m, the thickness is 0.3 m, the height of the connecting beams is 0.5 m and the elasticity and shear moduli of the structure are $E = 2.85 \cdot 10^6$ kN/m^2 and $G =$ 1055556 kN/m^2, respectively.

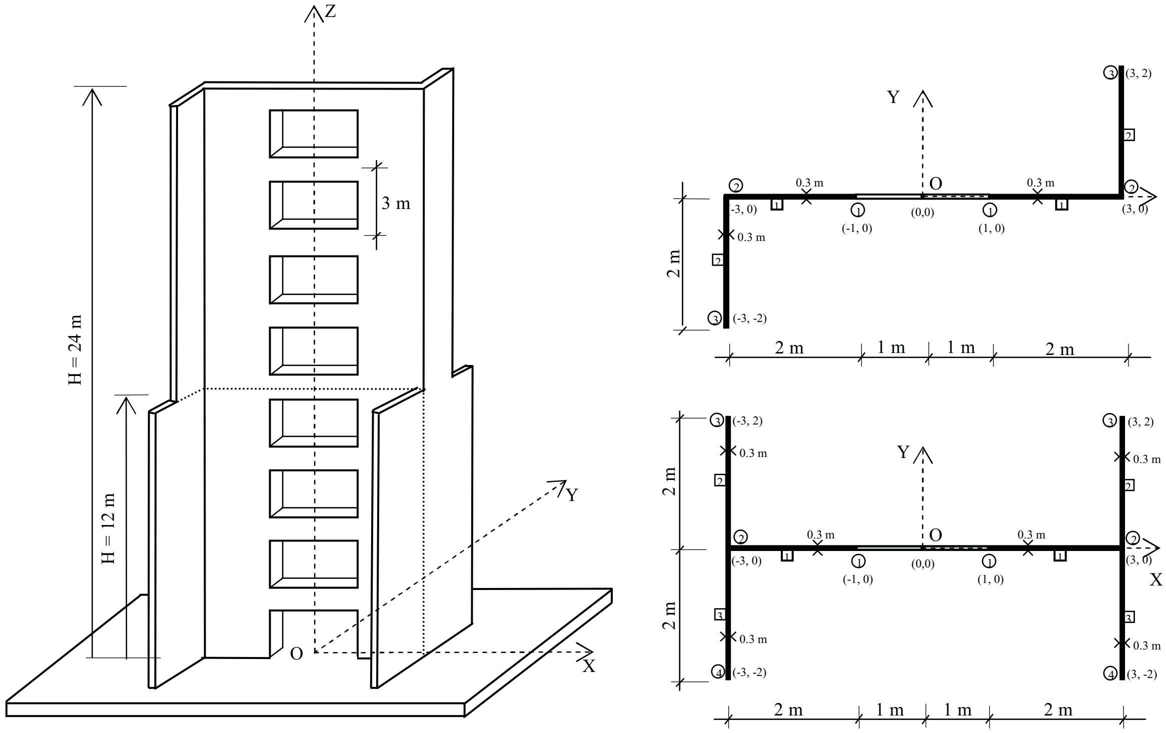

Figure 3. 3-D view and cross-sectional views of the structure

According to the lumped mass assumption, the lumped masses, which were calculated by the computer program, were concentrated at the center of the whole cross-sectional area of the structure.

Table 1 compares the natural frequencies found by the program prepared in the present work and the SAP2000 structural analysis program, expressing the percentage differences.

Mode	Present Study (CCM)	SAP2000 (Frame Method)	% difference
	Natural Frequencies	Natural Frequencies	
1	1,07270	1,07366	0,09
2	1,61807	1,60347	0,91
3	4,01622	4,01831	0,05
4	5,88496	5,85470	0,52
5	10,68451	10,63805	0,44
6	14,19554	14,14731	0,34
7	19,27886	19,19478	0,44
8	25,67233	25,44843	0,88
9	31,10287	30,91459	0,61
10	38,98560	38,45470	1,38
11	46,79325	46,35447	0,95
12	53,47172	52,34942	2,14
13	65,79978	64,26048	2,40
14	69,71574	67,71944	2,95
15	91,63290	88,45926	3,59
16	107,58726	103,86768	3,58

Table 1: The natural frequencies found by the CCM and SAP2000

Mode shapes in X and Y directions were compared by normalizing each with respect to the values at the top of the structure.
Figure 4 presents the mode shapes of the shear wall, found by the present program and the SAP2000 structural analysis program, both in the same table for the same quantity.

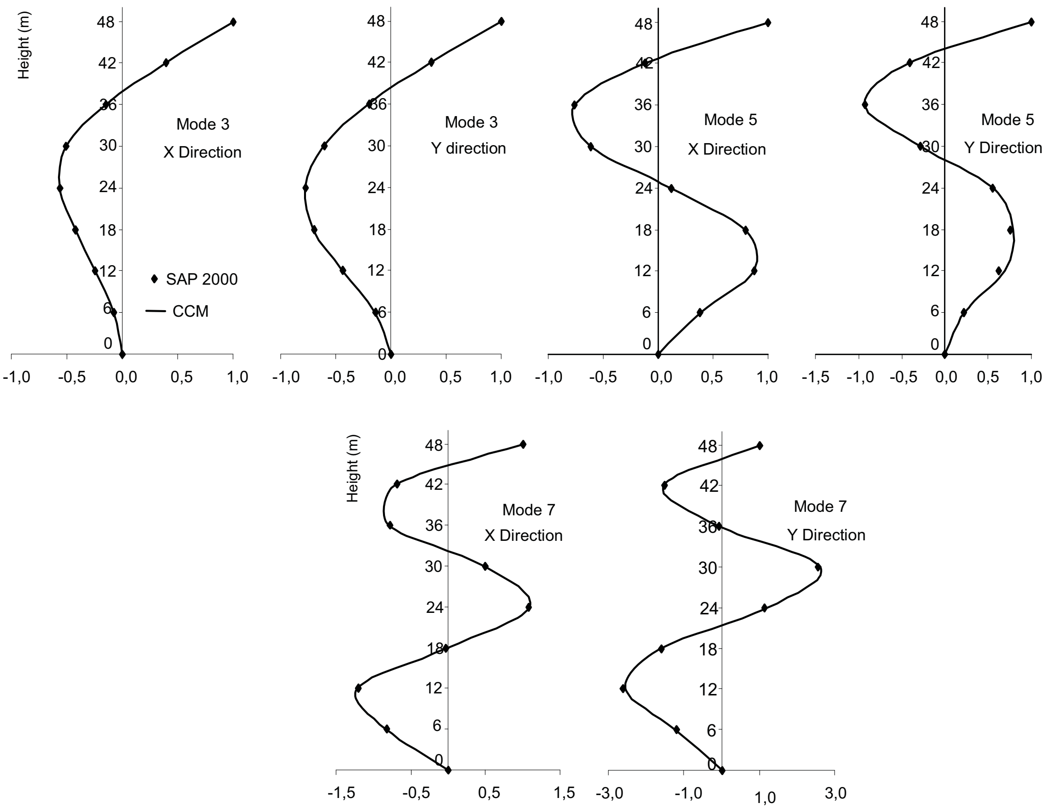

Figure 4. Comparison of third, fifth and seventh mode shapes found by both the present study and SAP2000

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