A QUANTISTIC MODELLING OF WINE EVALUATION IN OENOLOGY– PROBABILITY ANALYSIS

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Abstract. This paper describes a formalization of the essential organoleptic characteristics assessed in the wine evaluation process. A scalar quality metrics is associated and bound to 3 dimensions and 6 variables representing standard organoleptic wine features detected by mouth-tasting (visual-tasting and nose-tasting are not the subject of this work). The correlation existing between such wine characteristics is mathematically modelled by a matrix of operators' values acting on the corresponding variables. An algebraic notation is developed to express the multi-dimensional nature of the wine quality and to provide a measurement tool for the still subjective evaluation of a wine. Probability distributions are computed, or deduced from frequency distributions, for the values measured on a sample of over 100 wines in order to test the metrics performance in terms of phase-space and bias. The statistical meaning of the empiric distributions obtained by applying such a wine evaluation metrics is analysed by benchmarking them versus known theoretical mathematical conditions: this reverse-engineering of the metrics allows the factorization of the intrinsic metrics features from the effects due to the interaction with the "observer" and his preferences. Finally, the relation between the independent and correlated quantities in the evaluation of the wines is emphasized, and a conditional probability model is proposed.

1 Introduction

A metric for wine evaluation in the domain of Oenology is introduced and its mathematical aspects are studied in this paper. A main concept is that any metrics in Oenology [1] must be benchmarked from a mathematical and quantitative point of view [2], as it is the case for a numerical fit or for a pseudo-random generator. In particular, the objectives of this work are:

- To define a metrics for quantitative assessments of wine evaluations, including the implementation of the existing correlations between the variables expressing the organoleptic characteristics of the wine.
- To benchmark the performance of the metrics by analysing the statistical distributions of the results it produces on a sample of over 100 wines.
- To understand the nature and the mathematical meaning of the interaction of the "observer" (human evaluating the wine subjectively) with the system represented by the sample of wines being "measured".

A proper wine evaluation represents a multidimensional system [3], involving dependencies between different variables. Consequently the occurrence of the evaluations' results can be expressed by conditional probability relations. The corresponding equations are derived by modelling and renormalizing the probabilities according to the correlations.

2 Definitions

Let us define Q to be the scalar expressing the overall quality score of a wine and let us define it to assume the following values:

 $Q = [0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1.0; 1^*; 1^{**}; 1^{***}].$

The range of Q can also be transformed to an integers' space: Q = [-9; +3].

Let us define α , ϕ , π as three independent dimensions in the evaluation of wine characteristics, representing the following:

 $\alpha = Architecture$; $\phi = Finesse$; $\pi = Power$.

Each dimension holds two correlated signed directions and operators, identified as < (left), and as > (right), representing and valuing the specific organoleptic features (variables) of the wine:

α:	$<\alpha = Structured, Complex$	$\alpha > = Equilibrated$, Harmonious
φ:	$<\phi$ = Sec, Dry	$\phi > = Fined$, Aged
π:	$<\pi$ = Sensory, Intense	$\pi > = Bodied, Full$.

Let us define each operator to apply the values:

[+1; 0; -1; -2; -3], also noted as: [*; id; -; --; ---],

with the constraints defined by the correlations/vetoes set in Table 1. These also imply that the sum of the left/right operators on each dimension is contained in the range [-3; +1]. (Note: the notations "0" or "*id*" represent the identity operator).

\geq	1	0	-1	-2	-3
1		ok			
0	ok	ok	ok	ok	ok
-1		ok	ok	ok	
-2		ok	ok		
-3		ok			

 Table 1. Operators correlation: matrix of allowed " < left operator " values for each " > right operator value " (and viceversa).

3 Metrics

The wine organoleptic features can thus be treated as eigenstates with discrete eigenvalues to quantitatively measure their quality. For example, the $<\pi$ characteristics of a wine can map the incremental values in their range [-3,+1] to the usual scale of adjectives used in Oenology: "weak", "short", "light", "sensory", "intense". The same approach is valid for the other defined organoleptic variables and related operators' values (the complete "dictionary" is included in the Appendix). The defined wine evaluation procedure can be interpreted via a tasting operator bringing the wine system into the eigenstates corresponding to the wine organoleptic characteristics (the eigenvalues giving a measure for their quality), and with the eigenvalues of a given state putting limits to the possible outcomes of the measurements on the coupled observable.

The constraints on the operator values applied to each dimension allow a discretized sampling of the correlation existing between pairs of organoleptic features of the wine: for example, considering the submatrix [0,-3]x[0,-3] in Table 1 and in the Appendix for the case of the α dimension, it is evident how a very structured wine has more channels open to be unbalanced (to different extents) than a monotonic wine has; similarly, in the π dimension, a high alcohol content poses a more severe challenge to the wine body than what can be the case for lighter wines. A special case is given by the +1 values of the operators, for which a 0 of the coupled operator is required (excellence on one side requires the highest constraint from the other side). The recognition of correlations between variables is essential to avoid biases (e.g. accumulation points with respect to the average due to empty regions in the allowed phase space) in the data distribution estimators. Hence any multi-variable wine evaluation form [4] should be tested statistically by checking in these terms the results it produces.

As a consequence of the definitions seen previously and of the exclusion rules in Table 1, each of the three α, ϕ, π dimensions can assume all (and only all) the following 12 patterns:

Therefore the total number of allowed combinations supported by the metrics for expressing a wine evaluation is: $12^3 = 1728$.



Figure 1. Graphical notation: charts comparing tasted wines belonging to different 'appellations'. The 3 axis represent independent dimensions for wine evaluation. The organoleptic characteristics, which are on the opposite directions of each axis, are anticorrelated.

The sum/combination of the six operators on the three α, ϕ, π dimensions is contained in the range [-9; +3] and it coincides numerically with the Q value, thus binding a scalar measure to the underlying multidimensional wine system. Examples:

Q=0.7 [*
$$\alpha - \phi - \pi$$
-] Q=0.1 [$\alpha - - - \phi - \pi$ -] Q=1*** [$\alpha * * \phi \pi *$]

Such algebraic notation of the kind $[{}_{i}\alpha_{j}{}_{i}\phi_{j}{}_{i}\pi_{j}]$ maps directly to standard graphical representations, of which some examples are shown in Figure 1.

4 Metrics benchmarks

Q is distributed within a range of 13 values. The probability distribution of the Q scalar has been computed on a sample of 109 wines tasting [5] and is shown in Figure 2. It is observed that the peak and the median of the distribution are left-shifted with respect to the centre of the range.



Figure 2. Q probability distribution.

The Q value histogrammed for these 109 degustations is not statistically compatible with a flat random distribution, but it rather seems to follow a combinatorial distribution. The probability P of the empiric Q values could not possibly be constant; in facts if P(Q) = const, then P(Q) = 7.7% for all Q values (range of 13 values); in such a case, also P(Q=+3) = 7.7%; but $P(Q=+3) = 8 \cdot P(*X)^3$ (because there are 8 possible permutations for all the three dimensions to be in the status *X or X* to give Q=1***); the solution of that equation gives $P(*) \approx 21.5\%$ (which would bring *X and X* together to hold about 43% of the total P), i.e. an absurd requirement on the allowed operator patterns.

The middle values of the distribution result to be more probable. This may be for combinatorial reasons or it may be intrinsic in the wine subjective taste (it is evident that the extremes of the range represent exceptional cases, good or bad, while the central values represent more normal situations). In any case, one could even consider Q as the independent variable in a wine assessment (evaluated first), and α, ϕ, π could be constrained consequently: however, Q being a scalar, and the wine evaluation being a multidimensional problem involving different wine characteristics [6], it is implied that any value assigned to Q "averages" anyway over the different wine variables during the tasting; i.e. the same value of Q can derive from different wines configurations. Hence the Q distribution is expected to hold always a strong combinatorial component.

In facts, it has been seen how a wine evaluation is expressed by the configuration obtained by the valued operators applied on all the α,ϕ,π dimensions (e.g. [* α - ϕ -- π -]); it has also been mentioned that since each dimension can assume 12 patterns independently from the others, it follows that the metrics can support 12x12x12 = 1728 configurations, i.e. 1728 different combinations. Consequently, if the 12 patterns of operators allowed on each dimension would have the same probability to occur, then all 1728 configurations would have the same probabilities), and it would be possible to predict and compute theoretically the Q distribution: the probability of each Q value would depend on the number of α,ϕ,π configurations leading to that value in the [-9, +3] range. Such a computation leads to the results shown in Table 2.

0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1*	1**	1***
-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
64/1728	144/1728	204/1728	219/1728	270/1728	255/1728	206/1728	141/1728	114/1728	61/1728	30/1728	12/1728	8/1728
3,7%	8,3%	11,8%	12,7%	15,6%	14,8%	11,9%	8,2%	6,6%	3,5%	1,7%	0,7%	0,5%

Table 2. Theoretical Q distribution in case $12x12x12=1728 \alpha, \phi, \pi$ configurations would have the same probability to occur.

The qualitative agreement of these results with the shape, the maxima, and the asymptotic minima of the empirical Q distribution in Figure 1 appears to confirm the underlying combinatorial nature of the system: also the leftshift of the peak with respect to the range middle-point is evident. On the other hand, the significant quantitative differences of the distributions (the median of the data in Table 2 is shifted towards lower values with respect to the median of the empirical Q distribution, and their right-end tail is shorter and lower) confirm that the 1728 α,ϕ,π configurations seem to be not-equiprobable, hence the 12 operator patterns on each α,ϕ,π dimension would not be equiprobable.

In summary:

- There is evidence for the signature of the combinatorial nature underlying the Q metrics.
- The 1728 configurations appear to be not-equiprobable, hence the 12 patterns of possible operators per dimension would not have the same probability to occur.

In facts, it has been found that the Q distribution is peaked and has median at values lower than the middle of the Q range, however such shifts are smaller than what required by an equiprobable operators $P(_iX_j)$ distribution (thus the Q distribution has a trend towards "better wines"). The crucial question is why the 12 operator patterns are not equiprobable (i.e. why there is such a trend on Q):

- 1. It is conceivable that the metrics is not centred in the phase space, and that de-facto it behaves like a decimal system plus degenerated solutions above Q=1.
- 2. It is conceivable that the analysed wines sample is not fully random, and that with an infinite random sample $P(_iX_i)$ will be flat and $P(Q_k)$ will be fully combinatorial.

In addition, the role of the observer in the wine tasting operation has to be considered.

5 Data analysis and interpretation

In order to answer the questions raised in the previous section, it is convenient to analyse the results of the wine evaluations for each wine type ("appellation", e.g. AOC in France; or "denominazione", e.g. DOCG in Italy): it is legitimate to assume that different wine types do not need to produce identical Q distributions from the sets of wines they include in the various "appellations" and "denominazioni"; hence, the average of the Q values obtained from the wines of each "appellation" and "denominazione" is taken to represent the relative wine type; the obtained averages are histogrammed (one Q mean value for each wine type) as function of Q_k bin.

The list of wine types used in the sample of 109 wines [5] tasted for this analysis is expected to represent a rather standard sample, not too dependent on the observer's preferences, and it is expected to follow a distribution representative of the overall Q distribution; this allows the study of the weight, in terms of wines multiplicity M, found for each "appellation" or "denominazione" and, ultimately, given to each bin or sub-range of Q values. Table 3 shows the distribution of the mean Q values and shows that its median is comprised between 0.55 and 0.60; moreover it shows that 37% of the total 109 wines are belonging to the wine types before the median (Q mean value lower than the median), while 63% are belonging to the wine types after the median (Q mean value greater than the median).

Q mean	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.	1*	1**	1***
N types	1	3	7	7	4	8	6	4	4	4	1	0	0
M wines	1	5	10	10	8	21	10	9	18	15	2	0	0

Table 3. Distribution of the mean Q values obtained for the different types (appellation, denominazione) of wine. Multiplicity of tasted wines summed for all types falling in each Q-mean bin. The median is at the 2nd entry of the bin 0.6.

Therefore the sample of 109 wines is not uniform and its distribution represents the observer's trend to interact with wines closer to his taste. It is noted that the insight given by this kind of analysis allows equalization options a posteriori for the wine sampling, otherwise very unlikely to be realized [2]: in facts, every observer is faced not only with an extremely large spectrum of wines and related characteristics [3], but also with a remarkably large variance of wine performance for his taste as function of how many years after vintage the very same wine is tasted.

However, the most important result of this analysis is derived from the comparison of the distributions of, respectively, the mean Q value per wine type, the overall empirical Q on all wines (from Figure 2), and Q for the full combinatorial case (i.e. equiprobability of the 12 operator patterns). The relevant data are shown in Table 4.

	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1*	1**	1***
Equi-P	3,7%	8,3%	11,8%	12,7%	15,6%	14,8%	11,9%	8,2%	6,6%	3,5%	1,7%	0,7%	0,5%
Ntypes-P	2,0%	6,1%	14,3%	14,3%	8,2%	16,3%	12,2%	8,2%	8,2%	8,2%	2,0%	0,0%	0,0%
Exp-P	4,0%	5,0%	7,0%	10,0%	12,0%	14,0%	12,0%	10,0%	6,0%	6,0%	5,0%	4,0%	5,0%

 Table 4. Comparison of the distributions of, respectively:

Q for equi-probable 1728 α , ϕ , π configurations, Q mean for the wine types, and the experimental Q on all wines.

In facts, straightforward fits to the data tabulated above, as shown in Figure 3, show that the Q mean distribution on all considered wine types ("appellations", "denominazioni") tends closely to the full combinatorial case (the differences are explainable as residual observer's neglecting of not favourite or known wine types), consistently with the hypothesis of a more truly random sample.



Q for equi-probable 1728 α , ϕ , π configurations, Q mean for the wine types, and the experimental Q on all wines. The data show that the distribution of Q mean on the sampled types of wine tends towards the fully combinatorial case (though a small trend remains to test types of wine which one knows better or is more likely to appreciate positively).

It is concluded that:

- The shape of the scalar Q probability distribution is quantitatively determined by a pure combinatorial effect (due to the binding to a 6-dimensional system), superimposed by an observer's specific and not constant probability distribution of the 12 allowed operators patterns on each dimension, which represents a trend to taste (and consequently to include in the analysis sample) favourite wines.
- Such an observer's interference, with respect to a neutral random sampling of the wines to be evaluated by the metrics, has to be measured and characterised.
- Due to the required modelling of the correlations between pairs of the 6 dimensions of the wine system, the metrics is intrinsically unbiased in the phase space of the wine evaluations measurements (in facts, the case of wines total sampling with large statistics tends to the pure combinatorial condition, i.e. to the equiprobability of the allowed operators patterns on each dimension.

The observer-specific probability distribution derived (using the sample of 109 wine evaluations [5]) for the 12 different patterns of possible operators per dimension (α,ϕ,π) is tabulated in Table 5 and plotted in Figure 4.

P %	*	id	-	-,-		,-	
<α	7,3	16,5	12,8	10,1	8,3	3,7	2,8
α>	6,4	16,5	14,7	10,1	7,3	6,4	3,7
< φ	8,3	19,3	11,9	8,3	6,4	6,4	4,6
φ >	4,6	19,3	15,6	8,3	7,3	4,6	2,8
< π	8,3	19,3	13,8	11,0	6,4	5,5	2,8
π>	5,5	19,3	13,8	11,0	7,3	3,7	2,8
mean	6,7	18,3	13,8	9,8	7,2	5,0	3,2
fluct	2,6	4,3	3,7	3,1	2,7	2,2	1,8

Table 5. Probability distribution of the absolute valued configurations that can result for α , ϕ , π .

For $X=\alpha, \phi, \pi$, the notation used for the operators' values in the table header means the following:

X, X, -X, -X, -X, -X, -X, -X for the left operator <X; and $X^, X, X^-, -X^-, X^{--}$, X^{--} for the right operator X > .

Such a distribution is sampled 6 times via the organoleptic variables associated to α , ϕ , π (left and right operators) and obtained as their average, showing a consistent functional dependence. It should be noted that the average function shown in Figure 4 refers to operators patterns in abscissa for which a symmetric counterpart exists (except for X and -X-): these should be taken into account for computing correctly the total probability normalization. It is also noted that those probability distributions fluctuate about twice less than what allowed by a Poissonian statistics (computed for reference in the line *fluct* of Table 5), consistently with the systematic boundary conditions they have to obey to. The white trend-line in Figure 4, relative to the averaged (*mean*) data distribution, is the calibration function for the observer's interaction with the wine evaluation metrics benchmarking.

As mentioned previously, the analysis of the non-uniformity of the probability distribution in Table 5 or Figure 4 (equivalent to shifting the Q distribution away from the pure combinatorial shape) provides useful feedback on the sample of the selected wines from a statistical point of view: for example, it might indicate that the wines sample used has an average quality superior/inferior (for the observer) to a world-wide random selection [6].



Figure 4. Trend-lines relative to the observer-specific probability distributions for the allowed operators' patterns per dimension (α, ϕ, π), as tabulated in Table 5. The mean distribution (white thick line) represents the calibration function for the interaction of the observer with the benchmarking of the wine evaluation metrics.

Finally, questions such as why patterns like -X- vs. --X (and -X-- vs. X---) have different probability to occur, despite they contribute with the same number of minuses to the overall score for a wine, will be addressed in the following section: this is due to the inherent combinatory triggered by the particular correlation matrix defined.

6 Probability modelling

At the level of research interest, it remains to be studied the direct effect determined by the organoleptic variables correlations onto the probabilities of occurrence for the values applied by the left and right operators on the same α, ϕ, π dimension. This means analysing the probability of finding any of the values +1, 0, -1, -2, -3 (or *, id, -, --, ---) on a given side of α, ϕ, π , regardless what is present on the opposite side of each variable α, ϕ, π : such a probability can be expressed as P(i), with i = +1, 0, -1, -2, -3 (or i = *, id, -, --, ---). Hence by construction the following formula holds:

$$P(i) = \sum_{i} [P(_{i}X_{i})], i.e. P(i) = \sum_{i} [P(i \& j)],$$

where: $X = \alpha, \phi, \pi$; i, j = +1, 0, -1, -2, -3; and & stands for the logical AND.

By scanning the same sample of 109 wine evaluations [5] analysed so far, the P(i) distributions for the various dimensions are obtained empirically; they represent the probability distributions of valued operators applied to "a" side of α , ϕ , π . By averaging the results from all dimensions, the following data are produced and recorded

in Table 6:

P %	*	0	-		
mean	6,7	49,2	28,6	12,2	3,2

Table 6. Mean probability distribution of an operator value to appear at a given side of a variable, regardless (i.e. integrating probabilities) what happens on the opposite side of the variable.

If there would be no correlations between the values +1,0,-1,-2,-3 obtainable on the two sides of a given α,ϕ,π variable, the probability P of finding the patterns *X, X, -X, -X, --X, --X (and of course their symmetric cases) during the wines evaluation would be given by the products of the probabilities seen in Table 6. The resulting probability distributions for the configurations listed above already show that the probability of patterns such as -X- vs. X-- (and -X-- vs. X---) is not necessarily identical (just for combinatorial reasons); in facts, the following numbers would be found, apart from a constant multiplicative term of renormalization of the total probability:

$$P(*X) = P(X^*) = 6.7\% \cdot 49.2\% = 3.3\%; P(X) = 49.2\% \cdot 49.2\% = 24.2\%; P(-X) = P(X^-) = 28.6\% \cdot 49.2\% = 14.1\%;$$

$$P(-X-) = 28.6\% \cdot 28.6\% = 8.2\%$$
; $P(--X) = P(X--) = 12.2\% \cdot 49.2\% = 6.0\%$;

 $P(-X-) = P(-X-) = 12.2\% \cdot 28.6\% = 3.5\%$; $P(--X) = P(X--) = 3.2\% \cdot 49.2\% = 1.6\%$.

The most interesting challenge introducing the correlations comes from the difficulty that one gets an underdetermined system, when trying to derive mathematically the probability of the 12 operator patterns on the three dimensions by using the empirical knowledge of the P(i) recorded in Table 6: in facts the matrix in Table 1 does not contain the sufficient information. Therefore it is necessary to develop a model to express the results via conditional probabilities:

$$P(*X) = P(* \& id) = P(id) \cdot P(* | id); \dots P(-X-) = P(-- \& -) = P(-) \cdot P(-- | -); \dots etc. \text{ for all } 12 \text{ cases.}$$

In the present work, it is proposed to use a first order expansion of the probabilities renormalizations due to the exclusion constraints by Table 1. This gives the following results (which are compared below vs. the mean data of Table 5, reported in the right-aligned brackets) :

$$P(*X) = P(* \& id) = P(*) \cdot P(id) \cdot [1 - P(*) - P(-) - P(-)]^{-1} \cdot 1 = 6.70\% \quad (vs. 6.70\%)$$

$$P(X) = P(id \& id) = P(id) \cdot P(id) \cdot [1 - P(*) - P(-)] \cdot [1 - P(*) - P(-)] = 19.62\% \quad (vs. 18.35\%)$$

$$P(-X) = P(-\& id) = P(-) \cdot P(id) \cdot [1 - P(*) - P(--)] \cdot [1 - P(*) - P(--)]^{-1} = 14.07\%$$
(vs. 13.76%)

$$P(-X_{-}) = P(-X_{-}) = P(-) \cdot P(-) \cdot [1 - P(*) - P(--)]^{-1} \cdot [1 - P(*) - P(--)]^{-1} = 10.09\% \quad (vs. 9.82\%)$$

$$P(-X_{-}) = P(--\& -) = P(--) \cdot P(-) \cdot [1 - P(*) - P(--)]^{-1} \cdot [1 - P(*) - P(--) - P(--)]^{-1} = 4.98\%$$
(vs. 5.05%)

$$P(---X) = P(---\& id) = P(---) \cdot P(id) \cdot [1 - P(*) - P(-) - P(--) - P(---)]^{-1} \cdot 1 = 3.21\%$$
(vs. 3.21%)

Such a modelling gives satisfactory results, by matching the empirical mean values of Table 5 with a precision at the level of few percent (and fits the total normalization with a 1.5% precision, which is of the same order of magnitude as the numerical rounding and error propagation derived from the empiric data, which is for example 0.7% on the total normalization for the mean data in Table 5).

7 Conclusions

The essential organoleptic characteristics assessed in the wine evaluation process have been formalized via a scalar quality metrics associated and bound to 3 dimensions and 6 variables. The mathematical modelling of the existing correlations between the variables expressing such organoleptic features has been proven to play a key role in the performance of any wine quality metrics.

Consequently, a dedicated algebraic notation has been developed to express the multi-dimensional nature of the wine quality and to provide a suitable measurement tool. The use of an equivalent graphical notation has also been demonstrated.

The statistical distribution of the scalar wine-quality measure is quantitatively determined by a pure combinatorial effect (due to the binding to a 6-dimensional system), superimposed by the effect of an observer's specific sampling of the wines phase space. The mathematical meaning of the interaction of the "observer" (human evaluating the wine subjectively) with the system represented by the sample of wines being "measured" has been quantified and calibrated.

The relation between the independent and correlated quantities in the evaluation of the wines has been formalized via a conditional probability model, which has been shown to be precise at the few percent level.

8 Appendix

A complete mapping of the operators' values on α, ϕ, π to a standard scale of adjectives used in Oenology is included in the following "dictionary":

			α : acidic	
		-α : less structured & not equilibrated	α : saturating	
	α- : not structured & less equilibrated	-α- : less structured & less equilibrated	α- : unbalanced	
α: flat	α: monotonic	-α: simple	α: structured & equilibrated	*α : complex
			α* : harmonious	

			ф : sour	
		- \$: less sec & not fined	φ : tannic	
	ф- : not sec & less fined	- ф- : less sec & less fined	φ- : bitter	
ф : flabby	φ : mellow	- \$: rounded	∳: sec & fined	* \u00ed : dry
			φ* : aged	

			π : syrupy	
		-π: less sensory & not bodied	π : alcoholic	
	π- : not sensory & less bodied	-π- : less sensory & less bodied	π- : thin	
π : weak	π : short	-π : light	π: sensory & bodied	*π : intense
			π * : <i>full</i>	

9 References

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