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A DEVELOPMENT OF AN ANALYSIS FOR ENERGY AND FLUID CONVECTION USING THE THERMAL DISCRETE BOLTZMANN EQUATION WITH AN ANALYTICAL SOLUTION OF A RELAXATION EQUATION

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Summary. In this paper the new time integral method for the discrete Boltzmann equation (DBE) is introduced. It is based on the exact solution of the relaxation equation, and it is achieved to stable, good accurate and rapid calculating method for fluid simulation using DBE. To describe the effectiveness of this method, the rid-driven flow and the natural convection in a square cavity are calculated. The results of these problems have good accuracy to compare with the past researches. Then the numerical method is effective for the computation of fluid and temperature using DBE, and it is expected to apply the high Reynolds number flow field or the high Rayleigh number flow field.

Introduction. The lattice Boltzmann method (LBM) [1] is noticed as a new simulation method for fluids. Numerical experience using LBM is realized easily because of the simple analysis, and LBM is calculated simply for the complex fluid or complex boundary. The analysis of LBM is calculating the ideal particles to behave drift and collision. The drift term is solved by linear equations. The collision term is solved complex equations but it is linealized by lattice Bhatnagar-Gross-Krook (BGK) model [1]. The density, momentum, and pressure of fluids are calculated by the velocity moment of the distribution function of the density. LBM has a disadvantage for numerical simulation methods using Navier-Stokes equation. The most disadvantage of LBM is only used for regular lattices. This is meaning no realization of a numerical experience using body fitted boundaries. This problem is solved by the characteristic Galerkin method by Lee et al. [6] or the finite volume method by Succi et al. [5] for the discrete Boltzmann equation (DBE). DBE is the basic equation of LBM. Their works are completed of the problem of LBM. But the method using the new models caused a new problem. This is about stability for the collision term. On LBM, the numerical stability is completely stable if the single relaxation time is calculated from the fluid parameter. But on DBE, it makes unstable region to depend on the time discretization method. On the analysis of the Navier-Stokes equation, there are two approaches solving the instability problem. One is the addition to the stability term, and another is calculating the exact solution of the differential equation. The balanced tensor diffuser (BTD) term is as an example of the stability term, and the cubic-polynomial interpolation (CIP) method is one of the applications using the exact solution of the advection term of the Navier-Stokes equation.

On this paper, the analytical formulation of the collision equation is introduced. It is named the

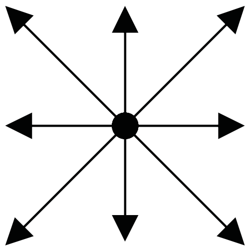

Fig. 1. The velocity vector of the ideal particles on 2D9V model.

Semi-Analytical method. First, the isothermal DBE is solved. The rid-driven flow in a square cavity is chosen. Then the thermal DBE is solved. As the benchmark test, the natural convection in a square cavity is performed. Each problem is compared with the past researches, and the effectiveness is discussed.

Numerical Formulation. DBE is written as

$$\frac{\partial f_i}{\partial t} + \boldsymbol{c}_i \cdot \nabla f_i + \boldsymbol{a} \cdot \nabla_c f_i = -\frac{1}{\tau_f}\left(f_i - f_i^{\mathrm{eq}}\right) \tag{1}$$

$$\frac{\partial g_i}{\partial t} + \boldsymbol{c}_i \cdot \nabla g_i = -\frac{1}{\tau_g}\left(g_i - g_i^{\mathrm{eq}}\right) \tag{2}$$

f is the distribution function for fluid density, $\boldsymbol{c}$ is the particle speed, i is the particle direction for D2V9 model defined as $\boldsymbol{c}_1$=(0,0), $\boldsymbol{c}_{2,3}$=(±1,0), $\boldsymbol{c}_{4,5}$=(0,±1), $\boldsymbol{c}_{6,7,8,9}$=(±1,±1) in Fig. 1, $\boldsymbol{a}$ is the acceleration, τ_f is the single relaxation time for density. g is the distribution function for internal energy. τ_g is the single relaxation time for internal energy. The equation for g is based on He et al [3]. Here Boussinesq approximation is applied for thermal DBE, the third term is rewritten as

$$\boldsymbol{a} \cdot \nabla_c f_i = -\frac{\boldsymbol{G} \cdot (\boldsymbol{c}_i - \boldsymbol{u})}{RT} f_i^{\mathrm{eq}} \tag{3}$$

$$\boldsymbol{G} = \beta g_0 (T - T_m)\boldsymbol{j} \tag{4}$$

where β is the thermal expansion coefficient, g_0 is the acceleration due to gravity, T is the temperature and T_m is the base temperature. The density, momentum and internal energy is calculated as

$$\rho = \sum_i f_i \tag{5}$$

$$\rho \boldsymbol{u} = \sum_i f_i \boldsymbol{c}_i \tag{6}$$

$$\frac{\rho DRT}{2} = \sum_i g_i \tag{7}$$

where D is dimension and R is the gas constant. The equilibrium distribution function in Eq. 1 and 2 is calculated as

$$f_i^{\mathrm{eq}} = \rho w_i \left\{1 + 3(\boldsymbol{c}_i \cdot \boldsymbol{u}) + \frac{9}{2}(\boldsymbol{c}_i \cdot \boldsymbol{u})^2 - \frac{3}{2}(\boldsymbol{u} \cdot \boldsymbol{u})\right\} \tag{8}$$

$$g_i^{\mathrm{eq}} = \rho RT w_i \left\{1 + 3(\boldsymbol{c}_i \cdot \boldsymbol{u}) + \frac{9}{2}(\boldsymbol{c}_i \cdot \boldsymbol{u})^2 - \frac{3}{2}(\boldsymbol{u} \cdot \boldsymbol{u})\right\} \tag{9}$$

where w_1=4/9, w_i=1/9 for i=2,3,4,5, w_i =1/36 for i=6,7,8,9. The single relaxation time is led to the viscosity and the thermal diffusivity,

$$\upsilon = RT\tau_f \tag{10}$$

$$\alpha = RT\tau_g \tag{11}$$

Eq. 1 and 2 is discretized using the Semi-Analytical method,

$$\overline{M}_{\alpha\beta} f_{\beta i}^{n+1} = \overline{M}_{\alpha\beta} f_i^n - \left(\tau_f \boldsymbol{A}_{\alpha\beta} D_i f_i^n + M_{\alpha\beta} f_i^n - M_{\alpha\beta} f_i^{\mathrm{eq}} - \tau_f M_{\alpha\beta} F^n - \tau B_{\alpha\beta} f_i^n\right)\left\{1 - \exp\left(-\frac{\Delta t}{\tau_f}\right)\right\} \tag{12}$$

and

$$\overline{M}_{\alpha\beta} g_{\beta i}^{n+1} = \overline{M}_{\alpha\beta} g_i^n - \left(\tau_f \boldsymbol{A}_{\alpha\beta} D_i g_i^n + M_{\alpha\beta} g_i^n - M_{\alpha\beta} g_i^{\mathrm{eq}} - \tau B_{\alpha\beta} g_i^n\right)\left\{1 - \exp\left(-\frac{\Delta t}{\tau_f}\right)\right\}. \tag{13}$$

To Eq. 12 and 13, the finite element method (FEM) is applied,

$$\overline{M}_{\alpha\beta} f_{\beta i}^{n+1} = \overline{M}_{\alpha\beta} f_i^n - \left(\tau_f \boldsymbol{A}_{\alpha\beta} D_i f_i^n + M_{\alpha\beta} f_i^n - M_{\alpha\beta} f_i^{\mathrm{eq}} - \tau_f M_{\alpha\beta} F^n - \tau B_{\alpha\beta} f_i^n\right)\left\{1 - \exp\left(-\frac{\Delta t}{\tau_f}\right)\right\} \tag{14}$$

and

$$\overline{M}_{\alpha\beta} g_{\beta i}^{n+1} = \overline{M}_{\alpha\beta} g_i^n - \left(\tau_f \boldsymbol{A}_{\alpha\beta} D_i g_i^n + M_{\alpha\beta} g_i^n - M_{\alpha\beta} g_i^{\mathrm{eq}} - \tau B_{\alpha\beta} g_i^n\right)\left\{1 - \exp\left(-\frac{\Delta t}{\tau_f}\right)\right\}. \tag{15}$$

In Eq. 14 and 15, the coefficient matrixes are meant as the mass matrix, the advection matrix and BTD matrix. These are written as

$$M_{\alpha\beta} = \int_{\Omega^e} N_\alpha N_\beta d\Omega \tag{16}$$

$$\boldsymbol{A}_{\alpha\beta} = \int_{\Omega^e} N_\alpha \cdot \nabla N_\beta d\Omega \tag{17}$$

$$B_{\alpha\beta} = \frac{\Delta t}{2} \int_{\Omega^e} \nabla N_\alpha \cdot \nabla N_\beta d\Omega . \tag{18}$$

$\overline{M}$ is meant lumped mass matrix for calculation speed to be faster. N is meant the shape function, and it written as

$$N_\alpha = \frac{1}{4}(1 + \xi\xi_\alpha)(1 + \eta\eta_\alpha). \tag{19}$$

α is the nodal point.

Numerical Result. The rid-driven flow for isothermal problem [4] and the natural convection for thermal problem are chosen as the benchmark problem. Each experience is shown in Fig. 2 and 3. The analysis condition is shown in Table 1 and 2. The numerical results for the rid-driven flow are shown in Fig. 4, and the average Nusselt number on the hot wall is shown Table 3.These results are proved that the Semi-Analytical method is effective method for fluid and temperature field. For isothermal flow, Re=5000 is solved stably. Then, the high Rayleigh number flow field is expected

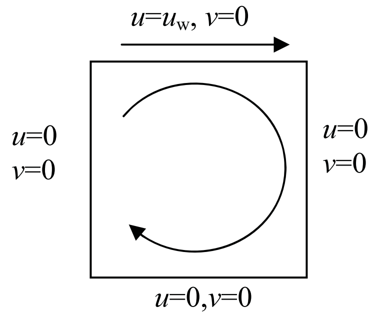

Fig. 2 The rid-driven flow.

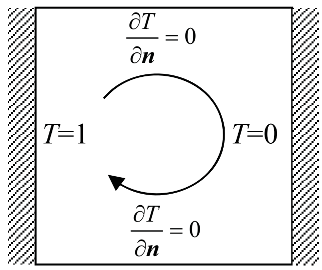

Fig.3 The Natural convection.

Table 2. The analysis conditions of the rid-driven flow.

Re	Lattice Number	Time Dif-ference	Wall Speed u_w
5000	257×257	0.005	0.1

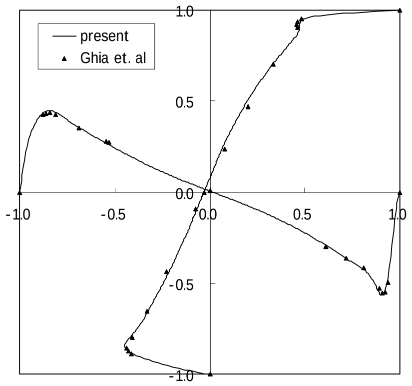

Fig. 4 The analysis result of the rid-driven flow.

Table 2. The analysis condition of the natural convection.

Ra	The Lattice Number	Δt
10^3	11×11	0.01
10^4	21×21	0.01
10^5	41×41	0.01

Table 3 The analysis result of the natural convection.

Ra	Present	Peng et al.[7]	de Vahl Davis[3]
10^3	1.116	1.117	1.116
10^4	2.295	2.235	2.234
10^5	4.563	4.511	4.510

to application to the Semi-Analytical method for the thermal DBE.

Conclusion. The Semi-Analytical method is suggested, and is applied to the rid-driven flow and natural convection. For the rid-driven flow, Re=5000 is solved stably, and the natural convection is also stability method for low Rayleigh number flow field. In the future, it is expected to the high Rayleigh number flow field and 3D problem.

References.

[1] S. Succi The Lattice Boltzmann Equation, *OXFORD*, (2001).
[2] X. He et al., *J. Comp. Phys*, **146**, pp282-300 (1998).
[3] de V. Davis, *Int. J. Numer. Meth. Fluid*, **3**(3) pp249 (1983).
[4] Ghia et al., J. Comp. Phys., **48**, 378-411(1982).
[5] S. Succi et al., Transport theory and Stat. Phys., **23**, 163-171(1994).
[6] T. Lee et al., J. Comp. Phys., **171**, 336-356(2001).
[7] Y. Peng et al., *Phys. Rev. E*, **68**, pp026701(2003).