ALGORITHMS OF CONSTRUCTION OF ADEQUATE MATHEMATICAL DESCRIPTION OF DYNAMIC SYSTEM

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Abstract. The main problem of mathematical modeling is the construction (synthesis) of mathematical model (MM) of motion of real dynamic system which in aggregate with model of external load (MEL) give the adequate to experimental observations the results of mathematical modeling. It was shown that the criterions of choice of good MM of dynamic system separately from choice of right MEL do not exist. Two basic approaches to this problem are selected. Within the framework of one of these approaches some algorithms are offered which allows receive adequate results of mathematical modeling. The different variants of choice model which are depending from final goals of mathematical modeling (modeling of given motion of system, different estimation of responses of dynamic system, modeling of best forecast of system motion, the most stable model to small change of initial data, unitary model) are considered. These problems are incorrect problems by their nature and so for their solution are being used the regularization methods. For increase the exactness of approximate solution the method of choice of special mathematical models was suggested. The test calculation was executed.

1. Introduction

At first in this paper the full process of synthesis of mathematical description (equations+model of external load) of real dynamic system and theirs calculations under some conditions will be called as mathematical modelling.
We shall be limited to consideration only by dynamic systems (processes) which are being described by the ordinary differential equations.
By the mathematical description of dynamic systems we understand the differential equations of motion which establish connection between the variables of state $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ... \tilde{x}_{n_1})^T$ of dynamic system (responses) and external load $\tilde{z}$ (input), $(\cdot)^T$ – a mark of transposition. For example, this connection for case of linear dynamic system has a form [1]:

$$\dot{\tilde{x}} = C_1 \tilde{x} + D_1 \tilde{z}, \qquad (1)$$

where C_1, D_1 – matrixes with constant coefficients.
Let's assume that external load $\tilde{z}$ and part of state variables $\tilde{x}_{r_1+1}, ..., \tilde{x}_{n_1}$, $r_1 + 1 < n_1$ are unknown. Other part of state variables in the equation (1) is measured by an experimental way. It is assumed that the equation of observation has a form:

$$\tilde{y} = F_1 \tilde{x},$$

where $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, ... \tilde{y}_{l_1})^T$, $F_1 = \{f_{ik}\}_{i=1,k=1}^{i=l_1,k=n_1}$ – matrix with constant coefficients of size $n_1 \times l_1$, thus $f_{ik} = 0$, for $k > r_1$. Let's consider in addition for simplicity that the matrix F_1 is diagonal $f_{kk} \neq 0$, for $k \leq r_1$, $f_{ik} = 0$, for $k \neq i$. If the part of external loads are known this case can be reduced to examined if using of linearity of dynamic system.
Let's consider known state variable $x_j(t)$ as two known internal loads $d_j \tilde{x}_j(t)$ and $[-d_j \tilde{x}_j(t)]$, $1 \leq j \leq r_1$, d_j - constant. Such interpretation of state variable allows to simplify initial system. Let's name such transformation by "j-section" of initial system [1].
In some cases after a lines of "j-sections" the initial system (1) will be transformed to some subsystem at which is known one state variable, for example, $\hat{x}_1(t) = \hat{x}(t)$ and all external loads $\hat{z}_k(t), k = 2, \ldots, m_2$, except $\hat{z}_1(t)$, for example, are known. This case is being reduced simply to a case when one external load $\hat{z}_1(t) = z(t)$ is not known only if using of linearity of a subsystem. Thus the received subsystem has a form:

$$\dot{\hat{x}} = C \hat{x} + D z, \qquad (2)$$

where $\hat{x} = (\hat{x}, \dot{\hat{x}}, .\ddot{\hat{x}}, .. \hat{x}^{(n-1)})^T$, $z = (z, \dot{z}, .\ddot{z}, .. z^{(m-1)})^T$, C, D – matrixes with constant coefficients.

With using of impulse transitive function it is possible to write down equality:

$$A_p\, z = \int_0^t K(t-\tau)\, z(\tau)\, d\tau = u(t) = B_1\, \hat{x},\ \hat{x} \in X\ , \tag{3}$$

where $K(t-\tau)$ – known kern, A_p is the operator of the certain structure, $A_p : Z \to U$; $B_1 : X \to U$.
If we return to old state variables the equation (3) will be transformed to a kind

$$A_p\, z = B_p\, x_\delta\ , \tag{4}$$

where $B_p\, x_\delta = B_1\, \hat{x} - B_{1,p}\, x_\delta$, $x_\delta = (\tilde{x}_1, ..., \tilde{x}_{r_1})^T$, $B_{1,p}$, B_p – operators translating elements $x_\delta \in X$ into U .
If with the help of a lines of "j-sections" it fails to allocate a subsystem (2) with one external load then the given reasonings lose the sense. If the initial dynamic system (1) has some unknown external loads and for each of them it is possible to receive subsystems such as (2) with one unknown external load, the above mentioned reasonings are valid, however further algorithms of construction of the adequate mathematical description essentially become more complicated.

2. Statement of a problem

For successful application of methods of mathematical modeling at research of dynamic systems it is necessary to execute synthesis of the mathematical description of real process which allows to receive results of mathematical modeling appropriate to experimental data [2]. Such result can be achieved by synthesis of "correct" mathematical model of motion of dynamic system and choice of "good" model of external load on this system, if system is open.
Let's illustrate it on an example of dynamic system with the concentrated parameters. The equation (4) we shall consider as basic. Let's assume, that the initial data $x_\delta = (\tilde{x}_1, ..., \tilde{x}_{r_1})^T$ are received by an experimental way with some known a priori by an error:

$$\left\| x_T - x_\delta \right\|_X \le \delta\ , \tag{5}$$

where x_T – exact initial data.
The check of adequacy to mathematical model of dynamic system and models of external influence in this case is reduced to check of performance of an inequality

$$\rho_U\,(A_p z, B_p x_\delta) \le \varepsilon\ , \tag{6}$$

where $\rho_U\,(\cdot,\cdot)$ there is a distance between elements of functional space U , ε – const, $\varepsilon > 0$, ε – required accuracy of concurrence with experiment. If the functional spaces are normalized then the inequality (6) can has a form

$$\left\| A_p z - B_p x_\delta \right\|_U \le \varepsilon\ . \tag{7}$$

It is natural that ε there can not be less size of δ .
Characteristic feature for examined problems is that the operator A_p is compact operator [4].
It is obvious that in the case of performance of inequality (6) operators A_p and function *z* are connected. It is easy to show that infinite set of various among themselves functions z which satisfy the inequality (6) exist at the fixed operator A_p in (6) [1,2,4]. And, on the contrary, at the fixed function z there are infinite many various operators A_p for which an inequality (6) is valid [1,2,4]. Thus, there are no opportunities of a choice of good mathematical model of dynamic system (or process) separately from a choice of correct MEL.
At research of concrete dynamic systems the structure of the mathematical description, as a rule, is fixed. However parameters of mathematical model it is necessary to believe given approximately. This error can be appreciated from above and, as a rule, she does not surpass 10 % [2].
Two basic approaches exist to problem of synthesis of couple MM and model of EL [2,3]:

1) MM is given a priori with inexact parameters and then MEL is determining for which the inequality (6) is valid [2];
2) Some MEL is given a priori and then is choosing MM for which the inequality (6) is satisfy [3].

By virtue of it the methods of identification of structure of mathematical model have the rather limited area of application.
At performance of concrete accounts it is necessary to take into account that the operators A_p , B_p , depend on a vector of parameters p of mathematical model of motion of dynamic system which are determined

approximately with some error. Thus we shall believe that for the normalized spaces Z, X, U the inequalities are carried out:

$$\| A_p - A_T \|_{Z\to U} \le h, \quad \| B_p - B_T \|_{X\to U} \le d, \tag{8}$$

where A_T, B_T – exact operators in the equation (8), h, d – known values.

3. Choice of value ε

If size ε in an inequality (6) to choose by a subjective way, the results of check of adequacy will be depended from the subjective factors. Therefore represents sense to design algorithms of check of adequacy, by which the size ε is determined by the objective factors.

It is obvious, that if the operators A_p, B_p will not change in future at mathematical modeling, in quality ε it is possible to take size $\| B_p \| \cdot \delta$. This conclusion follows from the estimation

$$\| A_p z_T - B_p x_\delta \| = \| B_p x_T - B_p x_\delta \| \le \| B_p \| \cdot \| x_T - x_\delta \| \le \| B_p \| \cdot \delta, \tag{9}$$

where $A_p z_T = B_p x_T$.

If to take into account an error of operators A_p, B_p, then in an inequality (6) the size of ε should be chosen on other algorithm. Let's assume that there are exact operators A_T, B_T, for which the inequalities (8) are valid and for which the equality is carried out

$$B_T x_T = u_T = A_T z_T,$$

where z_T – exact solution of the equation (4).

Then the estimation is fair

$$\| A_p z_T - u_\delta \|_U = \| A_T z_T - A_T x_T + A_p x_T - u_\delta \|_U \le \| A_T - A_p \|_U \| z_T \|_U + \| A_T z_T - u_\delta \|_U \le h \| z_T \|_Z + \| B_T x_T - u_\delta \|_U \le$$

$$\le h \| z_T \|_Z + \| B_T x_T - B_p x_T \|_U + \| B_p x_T - B_p x_\delta \|_U \le h \| z_T \|_Z + d \cdot \| x_T \|_X + \| B_p \| \cdot \delta .$$

Thus, it is possible to accept

$$\varepsilon = h \| z_T \|_Z + d \cdot \| x_T \|_X + \| B_p \| \cdot \delta . \tag{10}$$

The estimation (10) is objective, but too rough if to take into account that the sizes h, d can be calculated only, if the exact operators A_T, B_T are known. Besides the size $\| z_T \|_Z$ is not a priori known. Thus, the estimation (10) is not constructive. Though the size $\| x_T \|_X$ is easily estimated through known sizes $\| x_\delta \|_Z$ and δ:

$$\| x_T \|_X \le \| x_T - x_\delta + x_\delta \|_X \le \| x_T - x_\delta \|_X + \| x_\delta \| \le \delta + \| x_\delta \|.$$

4. Adequacy in nontraditional problems

Let's consider some nontraditional problems of synthesis of the adequate mathematical description within the framework of the first approach [1,2].

Let there z_p is a solution of an extreme problem:

$$\Omega[z_p] = \inf_{z \in Q_{\delta,p}} \Omega[z], \tag{11}$$

where $\Omega[z]$ – stabilizing quasi-monotonic functional [4],

$$Q_{\delta,p} = \{ z : \| A_p z - B_p x_\delta \| \le \| B_p \| \cdot \delta \}.$$

It is obvious, that any function from the set $Q_{\delta,p}$, switching z_p, will satisfy to a condition of adequacy (9).

In works [1,2] some nontraditional problems of construction of the adequate mathematical description are offered. For example, the following problem of definition of model of external load was examined within the framework of the first approach:

$$\inf_{A_p, B_p} \inf_{z \in Q_{\delta,p}} \Omega[z] = \inf_{A_p, B_p} \Omega[z_p] = \Omega[z_p^0]. \tag{12}$$

In this case estimation of adequacy is of the following form:

$$\inf_{A_p,B_p} \left\| A_p z_p^0 - B_p x_\delta \right\| = \inf_{A_p,B_p} \{ \left\| A_T - A_p \right\| \cdot \left\| z_p \right\| + \left\| A_p z_p - B_p x_\delta \right\| \} \le \left\| B^0 \right\| \cdot \delta \ ,$$

where $\left\| B^0 \right\| = \inf_{B_p} \left\| B_p \right\|$. Thus was used the property $\left\| A_p z_p - B_p x_\delta \right\| = \left\| B_p \right\| \cdot \delta$ of regularized solution with quasi-monotonic functionals [4].

Write the estimation of adequacy with the size ε as

$$\varepsilon = h \left\| z_p^1 \right\| + \left\| B^1 \right\| \cdot \delta \ , \tag{13}$$

for extreme problem

$$\sup_{A_p, B_p} \inf_{z \in Q_{\delta,p}} \Omega[z] = \sup_{A_p,B_p} \Omega[z_p] = \Omega[z_p^1] \ , \tag{14}$$

where $\left\| B^1 \right\| = \sup_{B_p} \left\| B_p \right\|$.

For extreme problem

$$\inf_{A_p,B_p} \left\| A_p z_p \right\| = \left\| A_{p^0} z_{p^0} \right\| \tag{15}$$

the estimation of adequacy has the form (6) with the size of ε as

$$\varepsilon = \left\| B^0 \right\| \cdot \delta \ . \tag{16}$$

The same result can be obtained for extreme problem

$$\sup_{A_p, B_p} \left\| A_p z_p \right\| = \left\| A_{p^1} z_{p^1} \right\| \tag{17}$$

with ε as

$$\varepsilon = \left\| B^1 \right\| \cdot \delta \ . \tag{18}$$

Thus, for various algorithms of construction of models of external loads exist the various objective estimations.
For synthesis of model of external load which gives good results of mathematical modeling uniformly for all possible operators A_p, B_p, the following extreme problem can being considered:

$$\inf_{z_p} \sup_{A_p,B_p} \left\| A_p z_p - B_p x_\delta \right\| = \left\| A_{p^u} z_p^u - B_{p^u} x_\delta \right\| \ .$$

In paper [1] some real accounts of models of external loads were executed which give adequate results of mathematical modeling.

5. Summary

The algorithms of synthesis of the mathematical description of real process (for example the motion of some dynamic system) are considered which allows receive adequate results of mathematical modeling. Two basic approaches to this problem are selected. Within the framework of one of these approaches some algorithms are offered.

6. References

[1] Menshikov Yu.L. Identification of Models of External Loads. // Proc. of ICINCO 2007, May 9-12, 2007, Angers, France, – p.376-379.

[2] Menshikov Yu.L., About adequacy of results of mathematical modeling. In: Proc. of Modeling–2008, May 16-18, Kiev, Ukraine, 2008, 119-124.

[3] Stepashko V.S., Method of critical dispersions as analytical apparatus of theory of inductive modeling. In: Problems of Control and Information, Kiev, Ukraine, 2008, 2, 8 - 26.

[4] Tikhonov, A.N., Arsenin, V.J. Methods of solution of incorrectly problems, Science, Moscow, 1979.