GAS INFRASTRUCTURE INVESTMENT: TWO SECTOR DYNAMIC OPTIMIZATION MODEL

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Abstract. This paper is devoted to modeling of growth of gas industry taking into account two possible technologies of gas delivery: via pipeline or LNG. It presents several model specifications that are analyzed using analytical and numerical methods.

1 Introduction

The topic about gas markets becomes a hot one. Contrary to oil, gas has a clear pattern of regional prices. That is why this market is much less competitive than one for oil, and producers (exporters) enjoy some level of market power even without cartel agreement. The reason for that is costly investment in pipelines. When the distance between producer and consumer is relatively small, investment in pipeline gives cheaper final price. Another technology (LNG) is liquidating gas, using relatively cheap trans-ocean transportation and de-liquifying it at the port of final consumer. Given that sea transport is cheaper than pipeline cost (per unit of distance), this gives more homogeneous pricing for liquid gas, and one can speak about the existence of world price for it. The paper presents dynamic optimization model about optimal investment path in two capitals related to gas-exporting industry.

1.1 Stylized Facts about Gas Industry

Gas industry is characterized by high concentration and high fixed costs. Three countries (Russia, Iran, Qatar) hold above 50 % of proved gas reserves. There exists two technologies of gas supply: by pipeline and liquefied natural gas (LNG). The pipeline technology does not produce any changes with gas, and transport cost is approximately proportional to the length of pipeline. LNG technology involves liquifying, transporting LNG at special vessels and the operation of reconversion into gas state at the port of arrival.

The typical cost of pipeline construction is 5-10 bln. USD. The costs are proportional to its length, and there are 4000 km long pipelines, connecting Siberia with Western Europe. The annual supply of gas along such pipeline can reach 30 bln. cub.m. According to Gas Technology Institute (GTI), construction of liquefaction plant with typical capacity of 8.2 mln.t costs 1.5-2 bln. USD. It is the most expensive component, with 30-45 % share of transporting costs. LNG receiving terminal costs less, 0.5-1 bln. USD, and LNG vessel costs 200-300 mln. USD. While shipping costs (in liquid state) depend on distance, they form only 10-30 % share of total transport costs.

Since pipeline gas can be delivered between a given set of of points in space, its price can vary across space substantially. Moreover, due to high investment cost in pipeline, there often exists local monopoly power for gas supplier. Only large consumers (like EU) can diversify the suppliers of pipeline gas, while small countries often have to accept price set by a monopolist. The situation with LNG is completely different. As we have seen, the distance between buyer and seller does not play important role here. In some sense, we can suggest the existence of world price for this gas. However, there is no reason for convergence of this price to the price of pipeline gas. First of all, this convergence is impossible in land-locked countries (like Austria), who have to pay additional cost to bring LNG from the closest equipped sea port. Second, there may exist spacial pattern of prices for pipeline gas, that takes distance into account.

There is an evidence about spatial and temporal variation of gas prices. The prices of LNG in Japan always has some premium in comparison with EU gas prices (mostly supplied by pipeline) between 1985 and 2004, but this premium varied from 10 to 40 %.

The role of gas in increasing in the world, and there is a positive trend in consumption demand (about 2-3 % per year). The demand for exports may grow even faster. It may be related to several factors: a) exhausting of resources in the countries that have been mainly self-suppliers in the past (like the USA); b) shift from using coal to using gas in electricity production due to ecological considerations; c) development of local gas networks in the countries that use gas for heating of houses.

1.2 Possible Economic Formalizations

It is possible to build many formal models, that reflect the stylized facts discussed above. Here is the summary of these stylized facts:
1. The cost of pipeline gas depends positively to the length of pipeline;
2. Investment in pipelines increases the demand for pipeline gas;
3. LNG has little price variation that we can neglect;
4. Markets for LNG are much more competitive than markets for pipeline gas (where supplier often has at least local monopoly power);
5. While gas reserves are limited, at present it is not the thought about future but the capacity is the main constraint of supply; at present, as market grows through the investment in infrastructure.

We will try to use most of these facts as formalized assumptions. Still, a high variety of models can be suggested.

The open problem is about the interplay between the prices of pipeline gas and LNG. Only this factor calls for several alternative model specifications. Another choice is the level of competitiveness. In all three models below we consider the problem of monopolistic optimization. Here the owner of pipeline and LNG strategy is the same, and decisions are taken in dynamically optimal way. There exists a (local) monopoly in pipeline gas supply and perfect competition in LNG supply.

Model 1 (section 2) and model 2 (section 3) differ in the specification of externality effect of capital investment in pipeline network on the price of pipeline gas. The demand is assumed to be linear but the choke price depends linearly on capital (strong externality). The model 2 allows for analytical solution, but it is simple as we always have convergence to a unique steady state. Contrary, model 1 allows for two equilibria. Model 3 (section 4) rests on the assumption of unique price for LNG and pipeline gas but the externality effect is weaker, growing as square root.

2 The Model 1

Notations. The following notations are used throughout the model: p_L - export price of liquid gas (at port), p_P - export price of pipeline gas, Q_L - volume of exported liquid gas (depends on demand), Q_P - volume of exported pipeline gas, c_L - unit cost to produce and to export liquid gas (depends on capital stock), c_P - unit cost to produce and to export pipeline gas, I_L, I_P - investments (flow variable) in infrastructure for liquid and pipeline gas, K_L - capital stock for liquid gas (capacity of liquifying plants, K_P - capital stock for pipeline gas (length of pipeline), $\pi(t)$ - inter-temporal profit function, r - discount rate, p_0 - "normal" world price for gas.

2.1 Formulation of Dynamic Optimization Problem 1

It is assumed that gas exporting firm is a monopoly in its country, which can also behave as a monopolist is local region (when regional price for gas is not exceeding its world price, coinciding with the price of liquid gas) but takes the world price of liquid gas as given. Every time period t (infinitesimal in the model) the firm possesses capital stocks $K_L(t), K_P(t)$ and invests I_L, I_P in infrastructure. It also can decide about exporting price of pipeline gas taking into account capacity constraint (market accessibility within constructed pipeline and demand sensitivity to price.

Then exporting firm solves the following optimization problem:

$$\max_{I_L, I_P, p_P} \int_0^\infty \pi(t) e^{-rt} dt, \tag{1}$$

$$s.t. \qquad \dot{K}_L = I_L - \delta_L K_L, \tag{2}$$

$$\dot{K}_P = I_P - \delta_P K_P, \tag{3}$$

$$\pi(t) = p_L Q_L + p_P Q_P - c_L(K_L) Q_L - c_P(K_P) Q_P - F(I_L) - G(I_P), \tag{4}$$

$$0 \leq Q_L \leq a_L K_L, \tag{5}$$

$$0 \leq Q_P \leq a_P K_P. \tag{6}$$

The last constraint has economic sense: since p_L is world price for gas, any rational consumer would never pay for pipeline gas the price higher than for liquid gas. In some regions of the world the price for pipeline gas is cheaper, because of advantages to have relatively small distance between producer and consumer that allows to save on transport cost.

There are several additional assumptions that simplify the model mathematically but keep minimal complexity to get interesting results.

Normalization. Assume $a_L = 1, a_P = 1$. This leads to re-scaling of time variable, because K is the stock, while Q is the flow of other variable (gas supply per unit of time).

Dependence of variable cost for pipeline gas. Investment in pipeline implies an extension of pipeline length. This broadens the market, on one hand, but on the other hand, makes the average delivery distance higher, thus increasing the average cost (to pump gas, since long-term investment in pipeline is out of this model component). Assuming linearity, one can write:

$$c_P = c_0 + \nu K_P. \tag{7}$$

Demand for pipeline gas. Consider the following demand function:

$$Q_P = K_P(2 - p_P/p_0). \tag{8}$$

With such a function, demand depends negatively and linearly on gas price. Also, we get demand equal to capacity (corner solution) when $p_P = p_0$.

Investment cost. In dynamic optimization literature investment cost usually has quadratic term.[1] Economic justification for the quadratic term comes from network interaction service. For example, repair costs can grow more that proportionally to the length of pipeline, due to urgency, because any wrong element stops the work of the whole system. Formally,

$$F(I_L) = I_L + \beta I_L^2, \qquad G(I_P) = I_P + \gamma I_P^2. \tag{9}$$

We also make two additional assumptions:
(i) there is corner solution for export of liquid gas at the current capacity of corresponding plants, i.e. $Q_L = K_L$,
(ii) there is internal solution for the price of pipeline gas, i.e. $p_0 < \hat{p}_P < p_L$.

2.2 Optimization within period

The problem (1-6) can be separated into two subproblems. We can observe that the choice of optimal price p_P within time period depends only on the current values of states and thus can be considered separately from the dynamic optimization. Using (5) and (10), we can find the optimal price:

$$\partial\pi/\partial p_P = Q_P + (p_P - c_P)\partial Q_P/\partial p_P = 0, \tag{10}$$

$$\hat{p}_P = p_0 + c_P(K_P)/2. \tag{11}$$

2.3 Hamiltonian and F.O.C.

Now we can substitute the optimal price into expression for profits and write Hamiltonian. Then the inter-temporal expression for the profit is:

$$\pi(t) = (p_L - c_L)K_L + K_P P_2(K_P) - I_L - \beta I_L^2 - I_P - \gamma I_P^2, \tag{12}$$

$$P_2(K) \equiv (p_0 - c_0/2 - \nu K/2)(1 - c_0/(2p_0) - \nu K/(2p_0)). \tag{13}$$

The current value Hamiltonian is:

$$H = (p_L - c_L)K_L + K_P P_2(K_P) - I_L - \beta I_L^2 - I_P - \gamma I_P^2 + \lambda[I_L - \delta_L K_L] + \mu[I_P - \delta_P K_P]. \tag{14}$$

The first two first order conditions (differentiation w.r.t. controls) give linear relationships between controls and co-states (that will allow to eliminate either the controls or the co-states):

$$\lambda = 1 + 2\beta I_L, \qquad \mu = 1 + 2\gamma I_P. \tag{15}$$

The last two conditions become:

$$\dot{\lambda} = \lambda(r + \delta_L) - p_L + c_L, \tag{16}$$

$$\dot{\mu} = \mu(r + \delta_P) - P_2(K_P) - K_P P_2'(K_P). \tag{17}$$

[1] This allows to escape single arc condition after getting $H_u = 0$ among the f.o.c.

2.4 Isoclines and Equilibria

Using (21), for isoclines $\dot{K}_L = 0, \dot{K}_P = 0$ we get the following expressions:

$$\lambda = 1 + 2\beta\delta_L K_L, \qquad \mu = 1 + 2\gamma\delta_P K_P. \tag{18}$$

For two other isoclines $\dot{\lambda} = 0, \dot{\mu} = 0$ we get:

$$p_L = \lambda(r + \delta_L), \tag{19}$$

$$\mu(r + \delta_P) = P_2(K_P) + K_P P_2'(K_P), \tag{20}$$

where

$$P_2(K) = \frac{1}{4p_0}(2p_0 - c_0 - \nu K)^2, \qquad P_2'(K) = \frac{\nu}{2p_0}(\nu K + c_0 - 2p_0). \tag{21}$$

Excluding λ, we get the expression for the value of K_L in critical points:

$$\frac{p_L - c_L}{r + \delta_L} = 1 + 2\beta\delta_L K_L. \tag{22}$$

Now we can exclude μ to find the equation for critical points:

$$(1 + 2\gamma\delta_P K_P)(r + \delta_P) = \frac{1}{4p_0}(2p_0 - c_0 - \nu K_P)^2 + \frac{\nu}{2p_0}K_P(\nu K_P + c_0 - 2p_0). \tag{23}$$

This equation is quadratic and thus has two roots.

2.5 Stationary Values of Capital Stock

Let us multiply the equation (23) by $4p_0$ and introduce new notations:

$$2p_0 - c_0 \equiv b > 0; \qquad \nu K_P \equiv x; \qquad 2\gamma\delta_P(r + \delta_P)/\nu \equiv u > 0. \tag{24}$$

Then we have:

$$3x^2 - (4b + u)x + (b^2 - r - \delta_p) = 0, \tag{25}$$

$$D = (4b + u)^2 + 4(r + \delta_P - b^2) > 0, \tag{26}$$

$$x_{1,2} = (4b \pm \sqrt{D})/6. \tag{27}$$

Both roots are positive iff $b^2 > r + \delta_P$, or $(2p_0 - c_0)^2 < r + \delta_P$. Note that p_0 was introduced by formula (10) in the expression for the demand for pipeline gas and has the meaning of such price, where demand is equal to capacity.[2] The parameter c_0 is the constant cost component of gas delivery. In general, the delivery of unit volume of gas is higher than c_0. In normal conditions, gas price at which all capacity can be sold, should be above cost, i.e. $p_0 > c_0$. Hence, $2p_0 - c_0 > p_0 > 0$. Then, there exist an interval of small positive discounts r and depreciations δ_P, such that the necessary and sufficient conditions for two roots holds; this happens for $p_0 < 2p_0 - c_0 < \sqrt{r + \delta_p}$.

2.6 Numerical Results for Model 1

Example of numerical experiment. Consider the following parameter values: $p_0 = 1, p_L = 1, c_0 = 0.5, c_L = 0.7, \nu = 0.5, \delta_L = 0.1, \delta_p = 0.1, \beta = 1, \gamma = 1, r = 0.05$. Then there are two positive steady states: in both cases $K_L = 5, \lambda = 2$, while for the lower steady state $K_p = 0.435..., \mu = 1.086...$, and for upper $K_p = 5.58..., \mu = 2.012....$ The lower steady state is a saddle (eigenvalues 0.68... and -0.63...), while the upper is unstable spiral, with eigenvalues $0.025 \pm i0.658....$

3 Model 2: Changes in Demand and Cost

3.1 Differences with Model 1

This version of the model has two differences from the previous. First of all, we introduce increasing costs also in LNG: $c_L = c_0 + \sigma K_L$. This cost is similar to c_P, with the only difference $\sigma < \nu$. Economic justification is as follows: the cost of gas extraction is increasing with volume, and this cost is transferred into LNG cost, while in the pipeline case it is added to the growing cost of transportation per unit of gas.

[2] As was noted before, capital investment in pipelines means geographical expansion of markets. For simplicity, it grows proportionally to the investment in pipeline, or to the total length of pipelines.

The second difference is in the shape of demand for pipeline gas. Now it is assumed in the following form:

$$Q_P = aK_P - p_p. \tag{28}$$

We still assume that $Q_L = K_L$.

3.2 Optimization

Inter-temporal monopolistic price. Now we have for the optimal inter-temporal price $\hat{p}_P$:

$$Q_P + (p_P - c_P)\partial Q_P/\partial p_P = 0, \tag{29}$$

$$\hat{p}_P = [c_0 + (a+\nu)(K_P)]/2. \tag{30}$$

New Hamiltonian and F.O.C. The new Hamiltonian becomes:

$$H = p_L K_L - (c_0 + \sigma K_L)K_L + aK_P\hat{p}_P - (\hat{p}_P)^2 - (c_0 + \sigma K_P)aK_P + (c_0 + \sigma K_P)\hat{p}_P - I_L - \beta I_L^2 - I_P - \gamma I_P^2 + \lambda[I_L - \delta_L K_L] + \mu[I_P - \delta_P K_P]. \tag{31}$$

The derivatives w.r.t. controls are the same as in the Model 1, and we can again exclude controls $\lambda = 1+2\beta I_L, \mu = 1+2\gamma I_P$, staying with the system of 4 differential equations:

$$\dot{K}_L = \frac{\lambda - 1}{2\beta} - \delta_L K_L, \tag{32}$$

$$\dot{K}_P = \frac{\mu - 1}{2\gamma} - \delta_P K_P, \tag{33}$$

$$\dot{\lambda} = \lambda(r+\delta_L) - p_L + c_0 + 2\sigma K_L, \tag{34}$$

$$\dot{\mu} = (r+\delta_P)\mu - \frac{(\nu-a)c_0}{2} - \frac{(\nu-a)^2}{2}K_P. \tag{35}$$

3.3 Isoclines and Steady States

The system of 4 equations represent two pairs of non-interacting equations:

$$\dot{K}_L = 0 \Rightarrow \lambda = 1 + 2\beta\delta_L K_L, \tag{36}$$

$$\dot{K}_P = 0 \Rightarrow \mu = 1 + 2\gamma\delta_P K_P, \tag{37}$$

$$\dot{\lambda} = 0 \Rightarrow \lambda(r+\delta_L) = p_L - c_0 - 2\sigma K_L, \tag{38}$$

$$\dot{\mu} = 0 \Rightarrow \mu(r+\delta_L) = P_1(K_P). \tag{39}$$

That is why the phase space can be decomposed into two non-interacting sub-spaces, (K_L, λ) and (K_P, μ), with a possibility of graphical analysis. All isoclines are linear in our case. In the first subspace, (K_L, λ), the equilibrium point is always unique:

$$\lambda = 1 + 2\beta\delta_L K_L, \qquad \lambda = \frac{p_L - c_0}{r+\delta_L} - \frac{2\sigma}{r+\delta_L}K_L. \tag{40}$$

We have $\dot{K}_L > 0$, iff $\lambda > 1 + 2\beta\delta_L K_L$, and $\dot{\lambda} > 0$, iff $\lambda > (p_L - c_0 - 2\sigma K_L)/(r+\delta_L)$. The isocline $\dot{K}_L = 0$ is always upward-sloping (and starts at $(K_L = 0, \lambda = 1)$, while the isocline $\dot{\lambda} = 0$ is downward-sloping and crosses the axis λ at the point $\hat{\lambda} \equiv (p_L - c_0)/(r+\delta_L)$. If $\hat{\lambda} > 1$, there exists a unique internal equilibrium point $(\bar{K}_L, \bar{\lambda})$. Given the field behavior, it is likely to be a saddle point. In the opposite case, that happens for too low price, $p_L - c_0 < r + \delta_L$, the equilibrium point stays outside the economically meaningful region of the first quadrant.

In the second subspace, (K_P, μ), we get the following isoclines:

$$\mu = 1 + 2\gamma\delta_P K_P \qquad \mu = \frac{(\nu-a)c_0 + (\nu-a)^2 K_P}{2(r+\delta_P)}. \tag{41}$$

In this case, both isoclines have positive slope, and all depends on their intersections with axis μ and relative value of slopes. The field $\dot{\mu} > 0$, iff $\mu > \frac{(\nu-a)c_0+(\nu-a)^2K_P}{2(r+\delta_P)}$. The field $\dot{K}_P > 0$, iff $\mu > 1 + 2\gamma\delta_P K_P$.

The case (a) happens for $\nu < a + 2(r+\delta_P)/c_0$ and $\frac{(\nu-a)^2}{2(r+\delta_P)} > 2\gamma\delta_P$. Then there exists a unique internal equilibrium. The type of equilibrium point requires further study; it might be a saddle in some cases.

The case (b) occurs for $\nu > a$ and $(\nu - a)c_0 > 2(r+\delta_P)$. In this case there is no internal equilibrium.

The case (c) happens for $\nu < a$ when the isocline $\dot{\mu} = 0$ starts above the isocline $\dot{K}_P = 0$ and has higher slope. Hence, they never intersect in the first quadrant.

The case (d) happen when $\nu > a$, $(\nu - a)c_0 > 2(r+\delta_P)$ and $\frac{(\nu-a)^2}{2(r+\delta_P)} < 2\gamma\delta_P$. In this case there is a unique steady state in the first quadrant.

Economic discussion. Internal solution with $\bar{K}_L > 0$ means that it is optimal in the long run to have finite investment in LNG infrastructure. This happens, when $p_L > c_0 + r + \delta_L$. Economic interpretation is simple. If price of LNG should be high enough to make it profitable not only to sell, but also to invest. The conditions to have internal solution for pipeline infrastructure, $\bar{K}_P > 0$, requires to have either case (a),

$$a > \nu - 2(r+\delta_P)/c_0 \quad \textit{and} \quad (a-\nu)^2 > 4\gamma\delta_P(r+\delta_P), \tag{42}$$

or (d):

$$\nu > a \quad \textit{and} \quad \frac{4(r+\delta_P)^2}{c_0^2} < (\nu - a)^2 < 4\gamma\delta_P(r+\delta_P). \tag{43}$$

The case (a) requires to have sufficiently high value of parameter a and sufficiently low parameters ν, γ, r, δ_P. Economically this means that expansion of market with prolonging pipeline offsets the growing cost to service it, and this happens in the environment of moderate discount r, depreciation δ_P and investment cost γ. In the case (d) we have a "tiny" parameters area, when $\nu > a$, but the square of it is inside the interval defined by other parameters. For this case to happen, it is necessary (but not sufficient!) to have $r + \delta_P < \gamma\delta_P c_0^2$.

3.4 Corner Solutions for model 2

Corner solutions happen, when one of capital stocks equals to zero in equilibrium. There also exists a formal possibility to shut down the whole gas industry by having both stocks equal to zero.

Now we need to take into account the constraints of non-negative capital, $K_P \geq 0$ and $K_L \geq 0$. The new Hamiltonian can be introduced as $H_0 \equiv H + \xi K_P + \eta K_L$. The new f.o.c. involve Kuhn-Tucker type conditions. The first 6 static conditions are as follows:

$$\partial H_0/\partial I_L = 0. \Rightarrow \lambda = 1 + 2\beta I_L, \tag{44}$$
$$\partial H_0/\partial I_P = 0. \Rightarrow \mu = 1 + 2\gamma I_P, \tag{45}$$
$$\partial H_0/\partial \lambda = 0. \Rightarrow I_L = \delta_L K_L, \tag{46}$$
$$\partial H_0/\partial \mu = 0. \Rightarrow I_P = \delta_P K_P, \tag{47}$$
$$\xi \partial H_0/\partial \xi = 0 \Rightarrow \xi K_P = 0, \tag{48}$$
$$\eta \partial H_0/\partial \eta = 0 \Rightarrow \eta K_L = 0. \tag{49}$$

Since $\partial H_0/\partial K_L = p_L - c_0 - 2\sigma K_L - \lambda\delta_L + \eta$, the new dynamic equations for co-factors are:

$$\dot{\lambda} = (r+\delta_L)\lambda - p_L + c_0 + 2\sigma K_L - \eta, \tag{50}$$
$$\dot{\mu} = (r+\delta_P)\mu - \frac{(\nu-a)}{2}c_0 - \frac{1}{2}(\nu-a)^2 K_P - \xi. \tag{51}$$

The equations $\dot{K}_P = 0, \dot{K}_L = 0$ are as before.

Only LNG. Suppose that under certain parameter conditions it is optimal to have zero investment in pipelines, i.e. $K_P \equiv 0$. Then the isocline $\dot{K}_P = 0$ corresponds to $\mu = 1$. Given that, the evolutionary equation $\dot{\mu} = (r+\delta_P)\mu - 0.5(\nu-a)c_0 - \xi$ will generate another isocline:

$$\xi = 0.5(a-\nu)c_0 - r + \delta_P. \tag{52}$$

The evolution in (K_L, λ) subspace is not affected, and the equilibrium values $(\bar{K}, \bar{\lambda})$ are found from the same linear system as before:

$$\lambda = \lambda(r+\delta_L) - p_L + c_0 + 2\sigma K_L,$$
$$\dot{K}_L = \frac{\lambda - 1}{2\beta} - \delta_L K_L.$$

Therefore, the corner steady state is:

$$K_L = \bar{K}_L, \lambda = \bar{\lambda}, K_P = 0, \mu = 1, \eta = 0, \xi = 0.5(a-\nu)c_0 - r + \delta_P. \tag{53}$$

Only pipelines. If we set $K_L \equiv 0$, we get no effect on evolution in (K_P, μ) space, and the steady state with $\lambda = 1, \eta = c_0 - p_L + r + \delta_L, \xi = 0, \bar{\mu}, \bar{K}_P$ and evolution $\dot{\lambda} = (r + \delta_L)\lambda - p_L + c_0 + \eta$.

4 Model 3

The goal of this version of the model is to take into account three factors:
a) to have one price, $p_L = p_p = p$; the economic motivation of this fact is related to equalization of pipeline and LNG prices at the end market and assumes several symmetric producers and costless arbitrage (here spatial difference across prices is not focused);
b) to eliminate dependence of variable cost of LNG on installed capacity in sector and to replace it by diminishing returns from investment into built physical capacity; thus LNG sector has constant variable cost but decreasing returns to capital while pipeline sector has constant returns (measured in physical capacity of output) to capital but increasing variable costs w.r.t. capital (proxy to pipeline length);
c) to consider more general form of positive externality of pipeline network length on the demand for natural gas.

Formally, new assumptions are replacing old:

$$p_L = p_p \equiv p, \qquad p = p_0 + \phi(K_p) - \tau(K_p + K_L); \tag{54}$$
$$\dot{K}_L = I_L - \delta_L K_L, \qquad \dot{K}_p = I_p - \delta_p K_p; \tag{55}$$
$$c_p = c_0 + \nu K_p, \qquad c_L = const > c_0. \tag{56}$$

The dynamic optimization problem is set similarly (the only difference is that now price is taken as given and producer decides only about investments):

$$\max_{I_L, I_p} \int_0^\infty e^{-rt} \pi(t) dt \tag{57}$$
$$s.t. \qquad \dot{K}_L = I_L - \delta_L K_L, \tag{58}$$
$$\dot{K}_p = I_p - \delta_p K_p. \tag{59}$$

For the general case of $\phi(K_p$ we get the following dynamical system:

$$\begin{aligned}
\dot{\lambda} &= \lambda(r+\delta_L) - p_0 + c_L + 2\tau(K_L + K_p) - \phi(K_p), \\
\dot{\mu} &= \mu(r+\delta_p) - p_0 + c_0 + 2\tau K_L + 2(\tau+\nu)K_p - \phi(K_p) - (K_L + K_p)\phi'(K_p), \\
\dot{K}_L &= -\delta_L K_L + \frac{\lambda - 1}{2\beta}, \\
\dot{K}_p &= -\delta_p K_p + \frac{\mu - 1}{2\gamma}.
\end{aligned} \tag{60}$$

Further we focus on sub-case with externality in the form of square-root.

4.1 Reduction

In the case of medium demand externality $\phi(K_p) = K_p^{1/2}$, we get the reduction of dynamical system:

$$\begin{aligned}
\dot{\lambda} &= \lambda(r+\delta_L) - p_0 + c_L + 2\tau(K_L + K_p) - K_p^{1/2}, \\
\dot{\mu} &= \mu(r+\delta_p) - p_0 + c_0 + 2\tau K_L + 2(\tau+\nu)K_p - \frac{3}{2}K_p^{1/2} - \frac{1}{2}K_L K_p^{-1/2}, \\
\dot{K}_L &= -\delta_L K_L + \frac{\lambda - 1}{2\beta}, \\
\dot{K}_p &= -\delta_p K_p + \frac{\mu - 1}{2\gamma}.
\end{aligned} \tag{61}$$

For further analysis numerical methods have to be used.

Numerical example. For the parameter values $p_0 = 1, c_0 = 0.2, c_L = 0.3, \nu = 1, \tau = 0.6, \delta_L = \delta_p = 0.1, \beta = \gamma = 0.2, r = 0.1$), we get positive steady state, with values: $K_L = 0.533..., K_p = 0.418..., \lambda = 1.021..., \mu = 1.016....$ It was found that the change of parameter ν leads to structural change: the component K_L declines and becomes negative at $\nu = 0.2425....$ For example, for $\nu = 0.2$ (and other values as listed) the steady state becomes $K_L = -0.168..., K_p = 1.66..., \lambda = 0.993..., \mu = 1.066....$

5 Conclusions

1. Modeling of the problem of dynamically optimal investment by gas monopoly in different technologies of gas delivery represents a practically important problem that can give rise to interesting mathematical models. In the case of natural gas there is no unique world price, and temporal evolution of this prices depends also on the development of infrastructure. The model set ups differ in the assumptions about strength of positive externality from gas pipeline network development on the demand for gas and the link between prices for pipeline and liquified natural gas.
2. In one of specifications (model 2) the solution can be found analytically and there is always a unique steady state. Under certain parameters, there is convergence to a saddle with both positive values of capital. However, corner solutions, when only one sector is optimally developed, are also possible.
3. In the specification of model 1 (when both choke price and demand slope depend linearly on capital stock) the nonlinear interaction between capitals can lead to emergence of two internal steady states. However, under certain parameter values there is unique saddle point.
4. The shape of externality also plays the role. If it is weaker (square root in model 3), the system becomes more complex, but we still can have convergence to unique steady state in some cases.

6 Literature

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