SIMULATION MODEL OF CE108 COUPLED DRIVES APPARATUS

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Abstract. The paper deals with the model of CE108 Coupled Drives Apparatus created in MATLAB-SIMULINK. The model was created in purpose to test the chosen identification and control algorithms which aimed to be verified at CE108 Coupled Drives Apparatus in the laboratory. CE108 Coupled Drives Apparatus is manufactured by TecQuipment. The model was created in the form of Simulink block having two inputs and four outputs in accordance with the real apparatus. The model was verified by simulation. Detailed description of used blocks is provided as well.

1 Introduction

Creating mathematical models of real objects in purpose of modelling and simulation is one of the most important phases of identification and control of real processes. Firstly, let us mention some interesting works from several areas in the field of modelling. Juslin (1995) presented the tool for high level model development based on specification of physical mechanism called APROS. Rivera and Prasad (1998) published the identification algorithm appropriate for the identification of a nonlinear compressor performance model. Ferreti and Girelli (1999) developed new approach to dynamic modelling of tracked vehicles in agriculture. Meusburger et al. (2000) presented the modelling of the work water section of a power plant group, the mathematical model is based on the Saint-Venant equations and the nonlinear partial differential equations of one-dimensional pipe flow. Kugi et al. (2000) described the simulator useful for hydrostatic transmission for vehicular drive system. Lunze at al. (2001) proposed the supervisor in purpose to supervise the process by means of a hybrid model employing Petri net. In the field of economics, Chiarella et al. (2003) provided new results useful for Keynesian macro model. Chernousko (2005) described his experience with the modelling of snakes locomotion. Chai et al. (2006) published the results of analysis obtained at Duvbacken wastewater treatment plant in Sweden in purpose to create the exact biological wastewater model. Schlake et al. (2006) proposed the state space model of a direct methanol fuel cell allowing analysis and optimization of the fuel cell.

The paper is focused on CE108 Coupled Drives Apparatus. The model is based on the data measured by Perutka and Dolezel (2006). This apparatus simulates the workstation where the speed and tension of moving product are measured. Interesting theoretical introduction to the modelling of this apparatus is available at Hagadoorn and Readman.

2 Apparatus description

The CE108 Coupled Drives Apparatus is described in the manual provided by the producer [12]. During rewinding the material passes through the workstation where the speed and tension are measured, which are dependable to each other, and they are corrected by the speed of motors located before and after the workstation. This situation is modified for laboratory experiments where the flexible belt is fastened on three wheels. Speed of two wheels is directly proportional to the number of revolutions of the servo-motors, these wheels are fixed. Third wheel may move, because it is fixed on the moving jib which is hung on the spring, and therefore it simulates the workstation with the measurement of the tension and speed. Two servo-motors control the speed and tension of the belt and the speed of 3 wheels. Their speed corresponds to the control voltage 0-10 V, which is 0-3000 rpm

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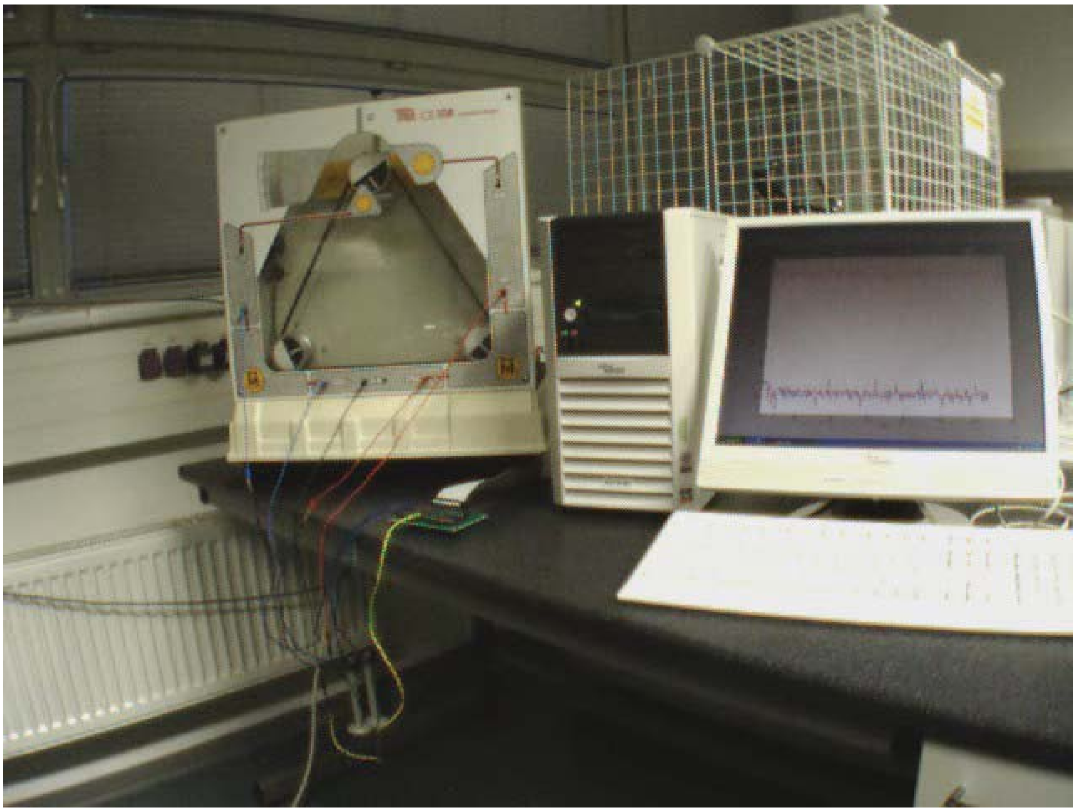

Figure 1. CE108 laboratory apparatus connected to PC

3 Theoretical background

The principal scheme of apparatus is shown in figure 2.

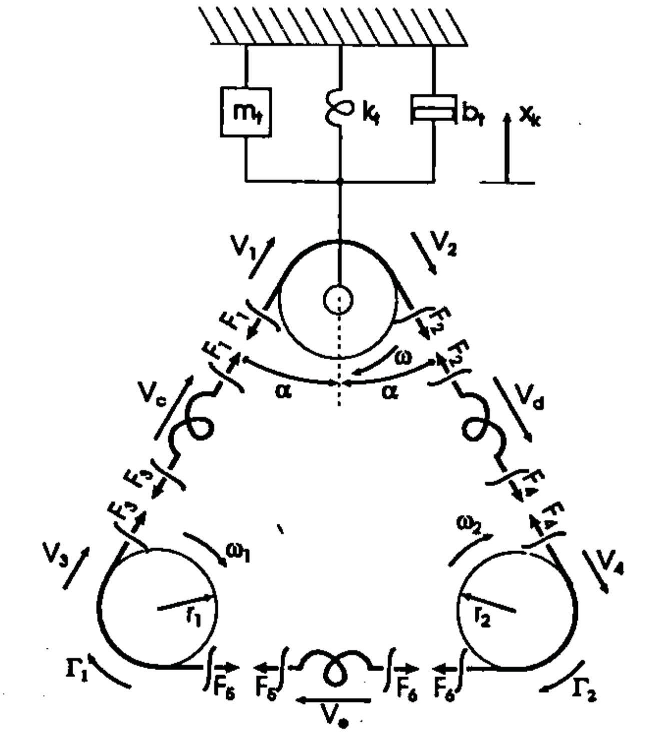

Figure 2. Scheme of CE108 laboratory apparatus

Let us suppose the both parts of the belt have the same angular deflection α from vertical line. Moreover, let us suppose the small change of turn angle of the moving jib in purpose to apply the distance x_k. The belt can be divided into linear parts of stiffness k_c, k_d and k_e, where wheel 1 and wheel 2 have inertia momentum and coefficient of rotation friction I_1, b_1, and I_2, b_2 respectively. The speed of belt parts is marked as v with the corresponding indexes. Let us view on the system as the one described in figure 1. Let us suppose no rotation friction of wheel proving $F_1 = F_2 = F$. Moving the forces on vertical line gives

$$F_t = 2\cos\alpha\ F$$

and the conservation law holds

$$\dot{x}_k F_t = F\left(v_1 - v_2\right).$$

Substition provides

$$\dot{x}_k\left(2\cos\alpha\right) = v_1 - v_2 \ .$$

The force F_t is given by $F_t = \dot{p} + k_t x_k + b_t.\dot{x}_k$, where p is the mass momentum m_t. The balance between torsion momentum of wheel 1 and 2 is described as

$$\Gamma_1 + F_3 r_1 - F_5 r_1 = \dot{h}_1 + b_1\omega_1 ,$$

$$\Gamma_2 + F_6 r_2 - F_4 r_2 = \dot{h}_2 + b_2\omega_2 ,$$

where h_1, h_2 are angular momentum $h_1 = I_1\omega_1$, $h_2 = I_2\omega_2$ and angular speed ω_1,ω_2 are given by $v_3 = \omega_1 r_1$, $v_4 = \omega_2 r_2$.

The balance of belt forces provides $F = k_c.x_c = k_d x_d$, $F' = k_e x_e$, where $F' = F_5 = F_6$ and $F = F_1 = F_2 = F_3 = F_4$ and x_c, x_d a x_e are the parts of the belt.

If the states of the system are given by h_1, h_2, x_c, x_e, x_k and p, it holds

$$\dot{h}_1 = \left[\frac{-b_1}{I_1}\right] h_1 + r_1 k_c x_c - r_1 k_e x_e + \Gamma_1$$

$$\dot{h}_2 = \left[\frac{-b_2}{I_2}\right] h_2 + r_2 k_c x_c - r_2 k_e x_e + \Gamma_2$$

$$\dot{x}_c = \left[1 + \frac{k_C}{k_d}\right]^{-1}\left[-h_1\frac{r_1}{I_1} + h_2\frac{r_2}{I_2} + p\frac{2\cos\alpha}{m_t}\right] \tag{1}$$

$$\dot{x}_e = \frac{r_1}{I_1}h_1 - \frac{r_2}{I_2}h_2$$

$$\dot{x}_k = \frac{p}{m_t}$$

$$\dot{p} = 2\cos\alpha k_c x_c - k_t x_k - \frac{b_t}{m_t}p$$

These state equations (1) provide the source of all necessary information about the apparatus. The equations can be simplified by the generally known facts, the motors are the same therefore $I_1 = I_2 = I$, $b_1 = b_2 = b$. The wheels have the same radius, the parts of the belt are the same therefore $r_1 = r_2 = r$ and $k_c = k_d = k_e = k$. Using these equations gives $\dot{h}_1 + \dot{h}_2 = -\frac{b}{I}(h_1 + h_2) + \Gamma_1 + \Gamma_2$, from which the plant of the first subsystem is $\omega_1(s) + \omega_2(s) = \frac{1}{sI + b}\left[\Gamma_1(s) + \Gamma_2(s)\right]$, where s is the complex variable.

For the speed v_c it holds $v_c(s) = \left[\frac{s^2 m_t + s b_t + k_t}{\left(1 + \frac{k_c}{k_d}\right)\left(s^2 m_t + s b_t + k_t\right) - (2\cos\alpha)^2 k_c}\right] r\left[\omega_2(s) - \omega_1(s)\right]$,

and
$$v_c(s) = \left[\frac{s^2 m_t + s b_t + k_t}{2\left(s^2 m_t + s b_t + k_t\right) - (2\cos\alpha)^2 k_c}\right] r\left[\omega_2(s) - \omega_1(s)\right]$$

The angular velocity is described by

$$\omega_1(s) = \left[\frac{Is^2 + bs + k^1(s)}{(Is + b)\left(Is^2 + bs + 2k^1(s)\right)}\right]\Gamma_1(s) + \left[\frac{k^1(s)}{(Is + b)\left(Is^2 + bs + 2k^1(s)\right)}\right]\Gamma_2(s) \tag{2}$$

$$\omega_2(s) = \left[\frac{Is^2 + bs + k^1(s)}{(Is + b)\left(Is^2 + bs + 2k^1(s)\right)}\right]\Gamma_2(s) + \left[\frac{k^1(s)}{(Is + b)\left(Is^2 + bs + 2k^1(s)\right)}\right]\Gamma_1(s) \tag{3}$$

where

$$k^1(s) = r^2\left(k_c G_1(s) + k_e\right) = r^2 k\left(G_1(s) + 1\right)$$

and

$$G_1 = \frac{s^2 m_t + s b_t + k_t}{2\left(s^2 m_t + s b_t + k_t\right) - (2\cos\alpha)^2 k}.$$

Moreover $\omega_1(s) - \omega_2(s) = \frac{s}{Is^2 + bs + 2k^1(s)}\left[\Gamma_2(s) - \Gamma_1(s)\right]$.

The angular velocity of the free-to-rotate wheel can be written as a function of the angular velocity of two remaining wheels

$$\omega(s) = \omega_1(s) + G_1(s)\left[\omega_2(s) - \omega_1(s)\right] \tag{4}$$

Or using the torsion $$\omega(s)=\frac{1}{Is^2+bs+2k^1(s)}\left(\begin{array}{l}\left[\dfrac{Is^2+bs+k^1(s)}{Is+b}\right]\Gamma_1(s)+\left[\dfrac{k^1(s)}{Is+b}\right]\Gamma_2(s)+\\ +sG_1(s)\left[\Gamma_2(s)-\Gamma_1(s)\right]\end{array}\right) \tag{5}$$

For $k^1(s)$ it holds

$$k^1(s)=kr^2\left(1+\left[\frac{s^2m_t+sb_t+k_t}{2\left(s^2m_t+sb_t+k_t\right)-\left(2\cos\alpha\right)^2k}\right]\right).$$

If the retaining screw is connected, $k^1(s)$ is constant, so

$$k^1(s)\to k^1=r^2\left[k_c+\frac{k_c}{1+\dfrac{k_c}{k_d}}\right]$$

and supposing $k_c=k_d$ gives

$$k^1=\frac{3}{2}kr^2,$$

where the right part of the equation is the effective torsion stiffness of the belt. The plant giving the change of the moving parts comparing to the inertia momentum is in the form

$$x_k(s)=\frac{2rk_c\cos\alpha\left(\Gamma_2(s)-\Gamma_1(s)\right)}{\left[\left(1+\dfrac{k_c}{k_d}\right)\left(s^2m_t+sb_t+k_t\right)-\left(2\cos\alpha\right)^2k_c\right]\left[Is^2+bs+2k^1(s)\right]},$$

which can be simplified as

$$x_k(s)=\frac{2rk_c\cos\alpha\left(\Gamma_2(s)-\Gamma_1(s)\right)}{\left[2\left(s^2m_t+sb_t+k_t\right)-\left(2\cos\alpha\right)^2k_c\right]\left[Is^2+bs+2k^1(s)\right]} \tag{6}$$

The belt tension is counted according to the angular deflection of the flexible belt x_k. The apparatus can be described by equations (1), the angular velocity is given by (5). The interaction among the motor speed and inertia momentum is given by (2) and (3). Finally, the angular deflection is given by (6).

4 Simulation model

In SIMULINK, there was created the model simulating the dynamics of the apparatus. The model as well as the apparatus has two inputs and four outputs. Following equations provides the mathematical description of all outputs of the apparatus in the Laplace transformation

$$Y_1(s) = \frac{g_a.g_1.\left[\left(I.s^2 + b.s + k.r^2.G_1(s) + k.r^2 - I.s^2.G_1(s) - b.s.G_1(s)\right)\right]}{[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2].(I.s+b)}.U_1(s) + $$
$$+\frac{g_b.g_1.\left[\left(k.r^2.G_1(s) + k.r^2 + I.s^2.G_1(s) - b.s.G_1(s)\right)\right]}{[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2].(I.s+b)}.U_2(s)$$

$$Y_2(s) = -\frac{g_a.g_2.r\cdot k\cdot\cos\alpha}{[s^2.m_t + s.b_t + k_t - 2.k.(\cos\alpha)^2].[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2]}.U_1(s) +$$
$$+\frac{g_b.g_2.r\cdot k\cdot\cos\alpha}{[s^2.m_t + s.b_t + k_t - 2.k.(\cos\alpha)^2].[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2]}.U_2(s)$$

$$Y_3(s) = \frac{g_a.g_3.\left[I.s^2 + b.s + k.r^2.G_1(s) + k.r^2\right]}{[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2].(I.s+b)}.U_1(s) + \frac{g_b.g_3.\left[k.r^2.G_1(s) + k.r^2\right]}{[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2].(I.s+b)}.U_2(s)$$

$$Y_4(s) = \frac{g_a.g_4.\left[k.r^2.G_1(s) + k.r^2\right]}{[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2].(I.s+b)}.U_1(s) + \frac{g_b.g_4.\left[I.s^2 + b.s + k.r^2.G_1(s) + k.r^2\right]}{[I.s^2 + b.s + 2.k.r^2.G_1(s) + 2.k.r^2].(I.s+b)}.U_2(s)$$

where s is the complex variable of Laplace transformation, U_1, U_2 are system inputs, Y_1, Y_2, Y_3, Y_4 are system outputs. Input parameters are other variables. $G_1(s)$ is defined as

$$G_1(s) = \frac{s^2 m_t + s b_t + k_t}{2\left(s^2 m_t + s b_t + k_t\right) - \left(2\cos\alpha\right)^2 k}$$

In figure 3, there is the main Simulink block and figure 4 shows the internal structure of the CE108 Simulink block.

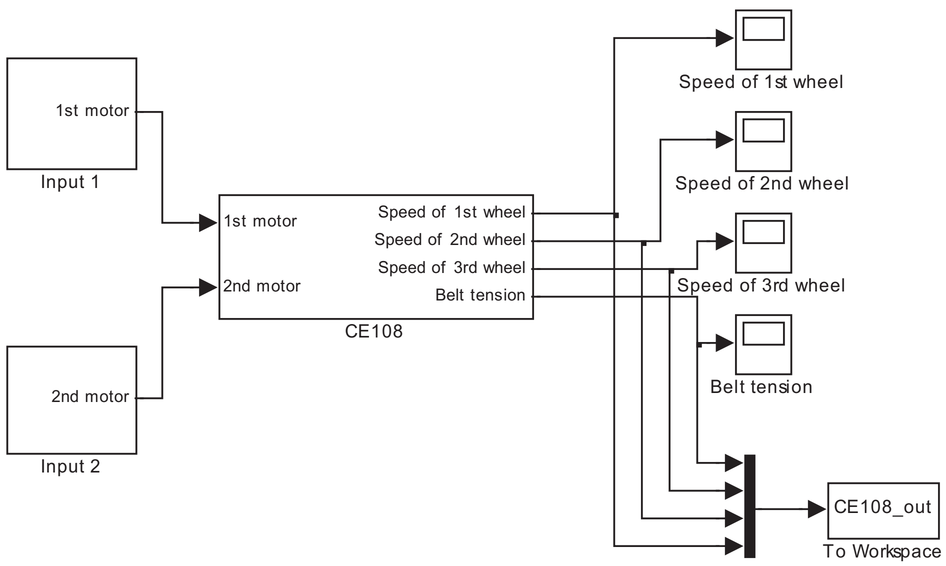

Figure 3. Block scheme of CE108 Simulink model

Figures 5 and 6 provides the visual comparison of the created model and the real system, CE 108 Coupled Drives Apparatus.

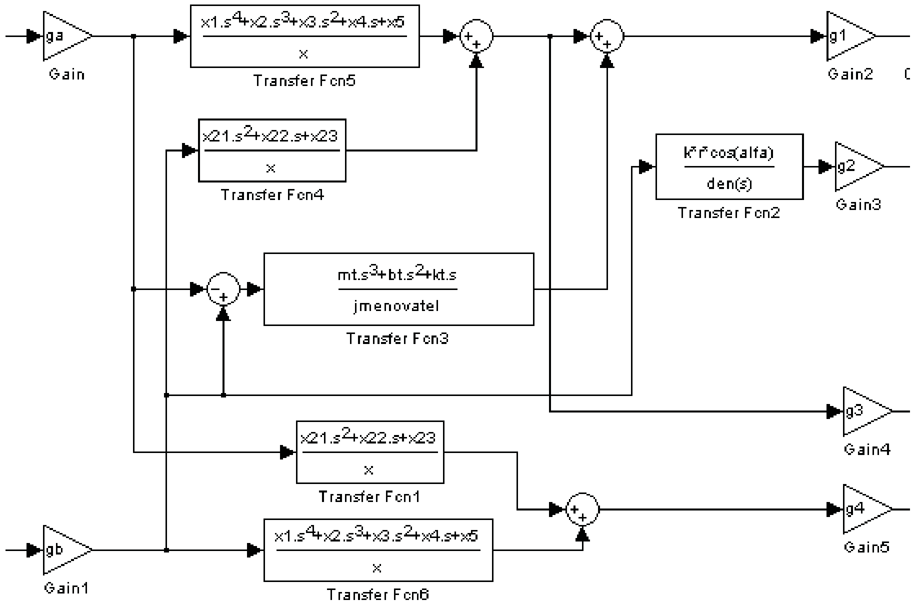

Figure 4. Internal structure of block scheme of CE108 Simulink model (without noise blocks)

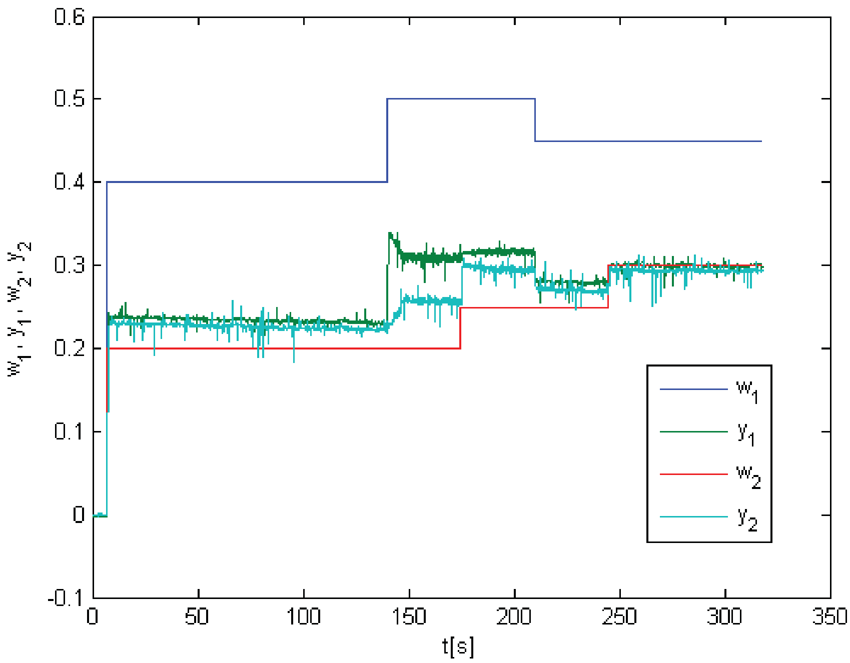

Figure 5. Response of CE108 Simulink model – 3rd wheel and belt tension

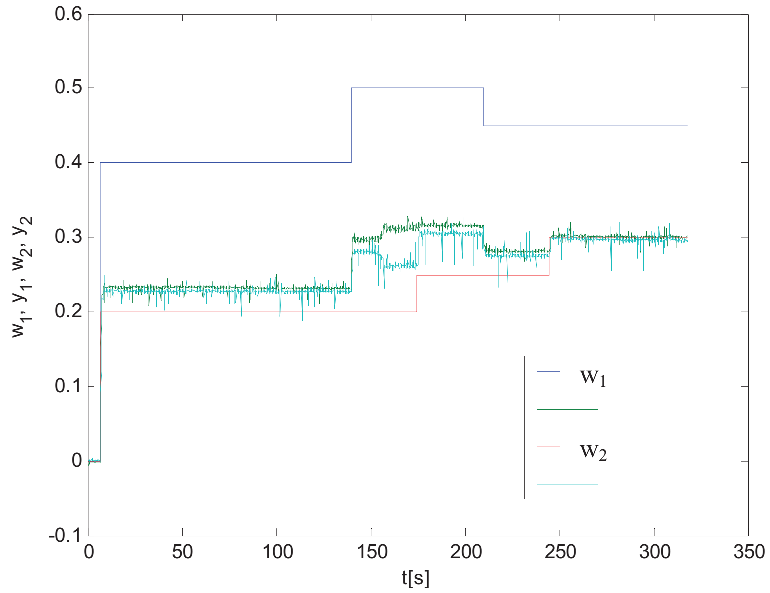

Figure 6. Response of CE108 – 3rd wheel and belt tension

5 Conclusion

The model of the real system in Simulink was created and it was verified that it can be used for simulation of control, which is useful before the real system is controlled. The future work will be directed to the verification of the proposed control on the simulation model before it is verified on the apparatus.

6 Acknowledgement

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