PROPERTIES OF THE NONSTATIONARY BALANCE-BASED MODELLING

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Abstract. In this paper, the simplified and nonstationary balance-based model is presented. The form of the model is unified and it results from the widely known idea of the mass or energy balance equation, which may contain the nonlinear terms describing the reactions or heat exchange/production phenomena. Usually, there is a large uncertainty on the detailed description of these terms and thus, in the suggested form of the model, they are represented by the only one time varying parameter, that must be estimated on-line. This approach is general and it benefits from the bilinear affine balance-based form. The estimation procedure always converges, which ensures high modelling accuracy. The model requires the feedback from the measurement data of the process output as well as of the disturbances and thus it is suitable only for the model-based control with the possible feedforward action, not for the quantitative considerations.

1 Introduction

In this paper, the simplified balance-based modelling for model-based control design is suggested. The idea benefits from the general and unified balance-based form applied for modelling biochemical processes for decades [1,2,3,7,10]:

rate of accumulation =(measurable) inflow – (measurable) outflow ± (unknown) R_Y, (1)

In the above model, the measurable signals are included as the *(measurable) inflow* and *outflow* and these terms represent the so-called balance-based part. The parameter R_Y is a positive or negative time-varying term, which represents one global reaction including all reversible and/or irreversible reactions or heat lost and/or production with nonlinear and unknown decryption. In other words, in the suggested form of the model (1) all the known and measurable quantities are included in the measurable terms while all the fluxes, for which the description must be considered as unknown, are represented by one time-varying parameter R_Y.

In the advanced control systems there are usually a number of measurable disturbances that provide the feedforward action. Actually, the feedback from the measurement data can be applied not only for this action, but also as a source of information for nonstationary modelling. In [7], the authors suggest the minimum modelling of the unknown terms based on the on-line estimation and observer design while in [8] the methods for the on-line model parameterization are presented. Both approaches lead to nonstationary modelling. The nonstationary approach presented in this paper is more general because it is based on the unified form of the simplified model with only one unknown parameter R_Y (representing the modelling inaccuracies and uncertainties) that must be estimated on-line.

2 General form of the balance-based model

The suggested model is based on the Eq. (1) and has the form of the first-order dynamic equation describing directly the process output Y(t), which can be chosen as one of state variables (a component concentration or the temperature) or as a combination of two or more state variables [4]. A process itself, taking place in a tank of time varying volume V(t) [m^3], can consist of a number of isothermal or nonisothermal biochemical reactions and/or heat production and lost phenomena with unknown description.

$$\frac{dY(t)}{dt} = \frac{1}{V(t)}\underline{F}^T(t)\underline{Y}_F(t) - R_Y(t) \tag{2}$$

The vector product $\underline{F}^T(t)\underline{Y}_F(t)$ represents the measurable terms of Eq. (1) while $R_Y(t)$ is a positive or negative time varying term and its description is assumed to be unknown. It represents all modelling inaccuracies and uncertainties. Let us note that Eq. (2) has a generalized form of a state equation describing Y(t) and, after simplification, it can be taken directly from a complete mathematical model of a process or easily derived by the general mass or energy balance considerations.

Eq. (2) can be a basis for the model-based controller under the following assumptions:

- the control variable must be chosen as one of the elements of the vectors $\underline{F}(t)$ or $\underline{Y}_F(t)$,
- the other elements of the vectors $\underline{F}(t)$ and $\underline{Y}_F(t)$ as well as the value of Y(t) must be measurable on-line at least at discrete moments of time or they should be known by choice of the user,
- the volume of a reactor tank V(t) must be measurable on-line or known if V = const.

These limitations are realistic because they allow for very large modelling uncertainty. They also ensure that Eq. (2) always has the bilinear affine form.

The simplified model (2) can be always satisfied because it is always possible to find the value of $R_Y(t)$ that satisfies this equation at the particular moment of time. Because the value of $R_Y(t)$ can vary in time, it is obvious that Eq. (2) can be satisfied at each moment of time by the appropriate choice of the value of $R_Y(t)$. The form of Eq. (2) ensures that there is always only one unknown parameter R_Y that must be estimated on-line to provide nonstationary modelling and to ensure high modelling accuracy. The estimation procedure is based on the discretised form of Eq. (2) with the backward first-order approximation of the time derivative of Y(t):

$$\gamma V_i \left(Y_i - Y_{i-1} \right) = T_R \underline{F}_i^T \underline{Y}_{F,i} - V_i T_R R_{Y,i} \,, \tag{3}$$

where i denotes the discretisation instant, T_R is the sampling time and $\gamma \in (0,1]$ represents the gain parameter, which allows for limiting the transient values of the approximation of the time derivative in the cases when the measurement data is noisy or when the system is strongly nonlinear with very fast dynamics. Then let us define the auxiliary variable y_i:

$$y_i = \gamma V_i \left(Y_i - Y_{i-1} \right) - T_R \underline{F}_i^T \underline{Y}_{F,i} \tag{4}$$

Combining Eqs. (3) and (4) allows for application of the well-known recursive least-squares (RLS) method with the forgetting factor α, which results in the following estimation procedure of the unknown value $R_{Y,i}$:

$$\hat{R}_{Y,i} = \hat{R}_{Y,i-1} - V_i T_R P_i \left(y_i + V_i T_R \hat{R}_{Y,i-1} \right), \qquad P_i = \frac{P_{i-1}}{\alpha} \left(1 - \frac{V_i^2 T_R^2 P_{i-1}}{\alpha + V_i^2 T_R^2 P_{i-1}} \right) \tag{5}$$

The scalar form of the estimation procedure results from the fact that there is always only one unknown parameter that is to be estimated. It ensures the convergence of the estimation without any additional excitation input signals that are usually necessary to guarantee the persistence of excitation for the on-line multiparameter identification [5]. In fact, even in the steady state the estimate $\hat{R}_Y$ always converges to its true value R_Y with the rate of convergence depending only on the value of the forgetting factor α [6].

3 Illustrative example

In the example vessel of constant volume V = 10 [L] (see, Fig. 1) there are two incoming flow rates: of cold water F_1 = 1 [L/h] and of hot water F_2 = 1 [L/h] with two corresponding inlet temperatures T_{in1} 20 [^{o}C] and T_{in2} = 80 [^{o}C]. The uncertainties in the system result from the unknown heat removal and from the fact that the outlet temperature T_{out} [^{o}C] is measured by the sensor located at the output of the pipe of length L = 10 [m] and of the cross section A = 0.0001 [m^2], which introduces significant transportation time delay $T_d = (L*A)/(F_1+F_2)$ [h] in the system. All the input quantities are assumed to be measurable on-line and the volume V is known. The modelled output is the outlet temperature $Y = T_{out}$, which is also measurable on-line.

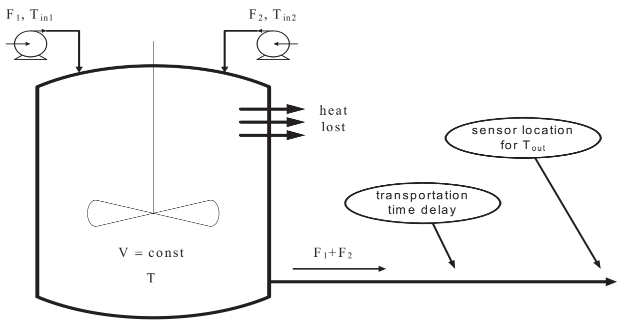

Figure 1. Simplified diagram of the example mixing vessel of cold and hot water

The general heat balance considerations lead to the dynamic equation presented below, in which the uncertainties resulting from the heat removal and from the not modelled transportation time delay, are represented by the unknown parameter R_Y:

$$\frac{dY(t)}{dt} = \frac{F_1(t)}{V} \left(T_{in1}(t) - Y(t) \right) + \frac{F_2(t)}{V} \left(T_{in2}(t) - Y(t) \right) - R_Y(t) \,. \tag{6}$$

Defining the vectors $\underline{F}(t) = \left[F_1(t) \;\; F_2(t) \right]^T$, $\underline{Y}_F(t) = \left[\left(T_{in1}(t) - Y(t) \right) \;\; \left(T_{in2}(t) - Y(t) \right) \right]^T$ shows that Eq. (6) has the form of the general Eq. (2) and it meets the requirements defined for that model.

The upper diagram of Fig. 1 shows the comparison between the variations of the output of the “true” system with the modelled heat lost and time delay and the output of the simplified model (6) with the on-line estimation carried out according to the estimation procedure (5). The lower diagram shows the corresponding variations of the “true” value of R_y calculated for the system with heat lost but without the time delay, which in the practice is unknown, in comparison with the estimated value of R_Y calculated for the simplified model (6). The experiment has been carried out according to the following scenario. The system is operated at the steady state and the estimation procedure (with the forgetting factor $\alpha = 0.1$ and with $\gamma = 1$) starts at t = 2. Then, at t = 5 the step change of the measurable inlet T_{in1} is applied. At t = 20 the bias error for the measurement of T_{in1} is applied, which does not influence the system but influences the model by the biased measurement of T_{in1}, introducing additional modelling inaccuracy. Finally, at t = 30 the flow rate F_1 is changed, which influences not only the outlet temperature but also decreasing the transportation time delay.

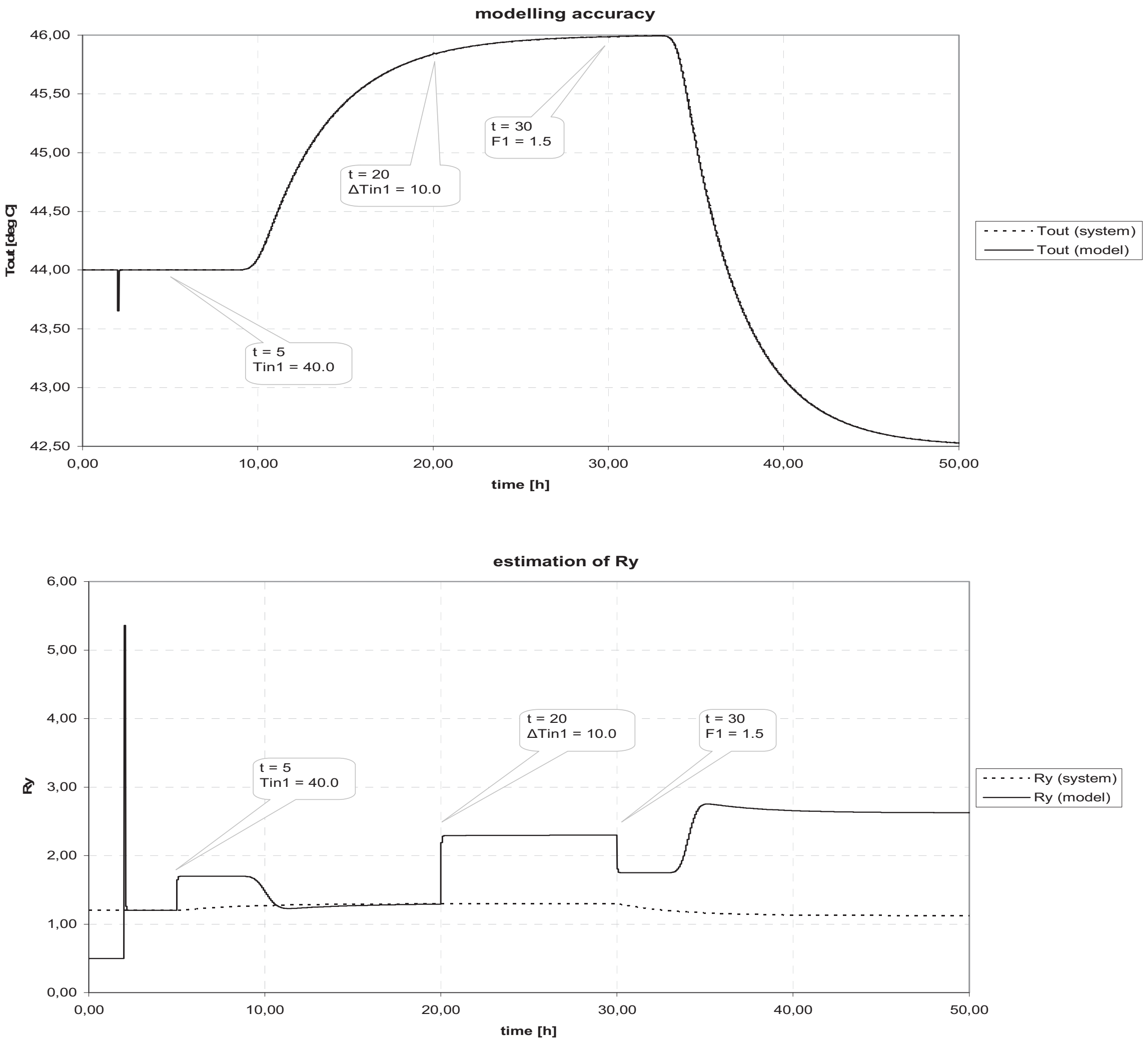

Figure 2. Modelling accuracy with corresponding estimation

Let us note that, despite of the very large modelling uncertainty, it is impossible to distinguish between the output of the “true” system and the output of its simplified model (6). Note that Eq. (6) does not include any description for the heat lost and for the significant time delay varying according to the changes of F_1 and equal T_d = 5 [h] at the steady state. The model (6) represents the first-order dynamic while the presence of the time delay significantly increases the relative order of the “true” system. This mismatch is compensated by the estimation procedure, which can be seen in the lower diagram of Fig. 2. After t = 5 the output of the model still tracks the output of the “true” system at the price of the bias between the “true” value of R_Y and its estimate. The bias disappears after the time delay, when the temperature in the vessel and the measured temperature at the outlet of the pipe T_{out} equalizes. At t = 20 the measurement error for the inlet temperature T_{in1} is introduced and the estimation procedure compensates for it at the price of the bias, which exists as long as the measurement error exists. In result, practically there is no influence of this error on the modelling accuracy. The variation of the time delay resulting from the change of the flow rate F_1 also has no practical influence on the modelling accuracy due to the compensating properties of the estimation procedure.

The experiment has been carried out without measurement noise, which allowed for the very small value of $\alpha = 0.1$ and for $\gamma = 1$. In the presence of the significant measurement noise there is a need to adjust higher value of α and smaller value of γ but it must be kept in mind that too large value of α increases the convergence time for the estimation procedure and thus it may degrade the modelling accuracy in transients.

4 Conclusions

In this paper, the unified and simplified form of the balance-based model has been presented. It always has the form of the first-order dynamic equation and it always includes only one unknown parameter, which represents all the modelling inaccuracies and uncertainties. This model is suitable only for model based control because it requires the feedback from the measurement data of the system output and of the measurable disturbances. The final form of the suggested model can be derived on the basis of the general well-known mass or energy balance law and thus it is very easy to understand for industrial engineers because it does not require the use of any sophisticated mathematics.

The unknown parameter must be estimated on-line but the scalar form of the RLS estimation procedure is very simple and also very general and unified. The on-line estimation ensures very good modelling accuracy despite of the very simple form of the suggested model. Namely, the model has the following features:

- It is resistant to the modelling inaccuracies resulting from the unknown description of the nonlinear terms representing the biochemical reactions or heat lost/production. The unknown terms can be included in the parameter R_Y and estimated on-line.
- The estimation procedure can also compensate for the mismatch between the order of the true system and the first order of the simplified model. The presence of the time delay does not degrade the modelling accuracy, even in the case when this time delay varies according to the changes of the operating point.
- In the practice, sometimes the sensor failure occurs, which may result in the constant indication or the measurement bias. As it was shown in the paper, the suggested simplified model can manage this problem without significant influence on the modelling accuracy, again due to the compensating properties of the estimation procedure.
- The model can be tuned for the case when the measurement noise is present. It requires the adjustment of the forgetting factor α and of the parameter γ. However, the practical advice is to keep the value of α as small as possible to ensure possibly the smaller convergence time for the estimation procedure.

The model can stand as a basis for the model-based control strategies, for example as PMBC [9], B-BAC [4,6]. The most important feature of the suggested model if the fact that modelling at the steady state is always very accurate, even if the modelling of the transients may be slightly inaccurate. Namely, there is no bias between the output of the system and the output of the model, which ensures that in the case of the model-based control there is no regulation offset and thus there is no need to include the integration in the final form of the control law.

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5 References

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