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MODELING EXERCISING ON WRIST EXERCISER

B. Curk[1] and P. Cafuta[1]
[1] University of Maribor , FERI

Corresponding Author: P. Cafuta, University of Maribor , FERI, 2000 Maribor, Smetanova 17, Slovenia;
peter.cafuta@uni-mb.si

Abstract. Powerball® is commercial name of a gyroscopic device that is marketed as a wrist exercisers. The device has rotor with two unactuated DOFs and can be actuated with suitable motion of additional human or robot wrist axis. After initial rotor's spin, applying the properly torque about wrist axis lead to spin-up of the rotor. Finding this torque intuitively is easy job for most peoples, but not so easy for technical consideration for example in robotics.

The articles main contributions are dynamic models with considered friction effects. Working principles of this device in all three modes: free rotor and both modes of rotor rolling in the housing are described. The work introduces reduction to 1 DOF excitation, considering preliminary research results, which enables laboratory control experiment. Estimation of friction parameters is also discussed. Both, the simulation with animation and experimental results are presented.

1 Introduction

Wrist exerciser named Powerball® is gyroscopic device popular in the 90-ties. They are patent pended [1]. Rolling connection of the rotor and the device housing enables the spin-up of the rotor by the appropriate wrist rotations. This movement is accompanied with the torque vector reaction proportional to the square of the rotor spin. The rotor can reach the speeds up to 16000 rpm for the plastic version showed on Fig.1. and an astonishing 20000 rpm for the metal version.

Our scenario includes manipulating robot, which is able to spin-up the Powerball®. This gyroscopic device represents a dynamic load to the robot as is discussed in [2] and represents underactuated and nonholonomic load for the manipulator kinematic chain [3]. Spin-up of the wrist exerciser is based on rolling which depends on friction between the rotor shaft and the housing. Friction is neglecting or it became significant. It depends on rotor state and on the wrist rotation reaction normal force on the shaft housing connection. The model is of the ODE type and has uncontinuous right side. Therefore it is a nonholonomic, underactuated, and variable structure type system.

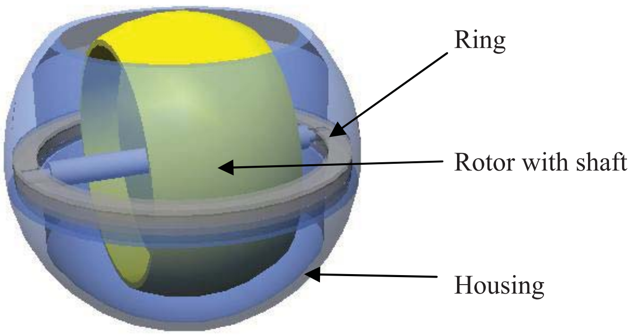

Figure 1. Wrist exerciser components.

The main contribution of the paper in comparison to the results in references [4,5,6] is the dynamic model which replicates all there modes of gyroscopic device: the free rotor, the left, and the right mode of rotor rolling in the housing. This model enables insight to dynamics which further serves in development of control experiment. It is of one active degree of freedom and the spin velocity output type. The parameter setting of the model and control are found by estimation and experiments.

The paper is organized as follows. First the modes of operation and appropriate models are discussed. The variable structure model is developed next. Then the robotic activation will be reported, control strategy discussed, and after that the experimental results are showed. On the end of the paper future work will be discussed.

2 Working principle

Wrist exerciser is essentially a gyroscope with at list three degrees of freedom in rotation. There are spin, precession, and nutation rotation. Powerball® is a guided gyroscope in nutation rotation with a special feature of rolling connection in precession and spin degrees of freedom. Nutational rotation causes precession torque reaction which: the first press spinning rotors shaft on a housing to roll in a same direction and the second precession torque acts on the rotor. So the first mode of operation is start-up to the minimal necessary spin. It can be done with a starting rope or some other starting device. The gyroscope after that only spin-off because there is friction in the bearings. The same is valid for Powerball® when the initial spin is not to low and independent from applied or not applied nutation rotation.

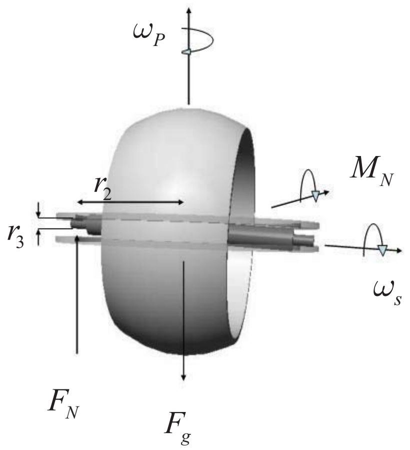

Figure 2. Rotor reaction on torque M_N .

When the spin is high enough two additional modes occur. Energy supply to the rotor is possible due to patent pended precession ring which enables direct connection between the rotor shaft and the housing of the Powerball®.

(1)

2.1 Coupling

With this rotor and housing connection the additional friction effect is observed . When the nutation reaction torque M_N is to low, only dissipation of energy is observed. But when the reaction torque causes normal forces on a connection in a degree that the friction is high enough, the rotor shaft begins to roll. The connection with the housing takes place in a gap so two connecting pairs are possible. One is up-down for left precession and another is down-up for opposite precession rotation. The situation for the force pair acting on the shaft is illustrated on Fig. 2.

When the rolling is without sliding the precession speed ω_p is proportional to the spin speed ω_s given with the equation (1). Direction of the precession depends on amount and direction of torque, which acts against the torque from gravitation on the rotor. In the worst case the rotor axes is horizontal and the equation (2) is valid. The ratio in diameters p of the shaft r_3 and the ring r_2 is given with equation (3) and reflects rolling velocities ratio.

$$\omega_P = \begin{cases} -1/p \cdot \omega_S;\ M_N \geq \mu \\ 1/p \cdot \omega_S;\ M_N \leq -\mu \end{cases}, \tag{1}$$

$$\mu \leq \mu_{\max} = \left| F_g \cdot r_2 \right|, \tag{2}$$

$$p = r_2 / r_3 \ . \tag{3}$$

Due to two possible rolling modes transition between them is not trivial. It takes place thru the free rotor mode. This transitions are called reversion and the rotor spin direction is not influenced. In both precession directions the same spin-up is possible.

After rolling takes place the wrist exerciser stays in this mode because the precession reacts on the nutation support. On this way the kinematic connection with the rotor take place. When no work is done to device this state will vanish due to the unavoidable dissipation in the system. But supplying the energy with rotation and the nutation torque thus adds to the rotor kinetic energy and the rotor spin-up. Precession rotor direction changes against nutation rotation, so the nutation should follow the rotor precession rotation.

This can be done by wobble the two device housing DOF in a human hand or with the robot wrist or only with rotational swing of the one housing DOF. Only the one DOF rotation simplifies the analysis as the experimenting setup so this is done in our work first. This simplified case incorporates singularity which must be avoided at the start-up and the transmitted energy is only one half of the full activated case.

The described principle is quite simplified. The detail discussion is possible only with modeling and the appropriate experiments. This will be done in next sections.

3 Dynamic model

The masses of housing and the ring are neglected as are the friction in bearings. Only the friction due to normal force F_N acting on the ends of rotor shaft will bi explicitly discussed. It conditioned left and right precession rolling and the free rotor spin.

The wrist exerciser has 2 DOF when fixed and additional DOFs then belong to the robot wrist.

The torque or impulse applied to rotating body changes its impulse $\mathbf{\Gamma}$:

$$\mathbf{M} = \frac{d}{dt}\mathbf{\Gamma} = \frac{d}{dt}\left(\mathbf{J}\cdot\boldsymbol{\omega}\right) \cdot \tag{4}$$

Where $\mathbf{J}$ is the inertia tensor and $\boldsymbol{\omega}$ the rotation velocity vector. The equation (4) holds only in the inertial coordinate system (CS). The entries of the $\mathbf{J}$ changes with the selection of coordinate system. The usual way to start modeling is in the coordinate system of the rotor. In our case the coordinate system of the ring is chosen. Thus the vectors and tensor components are most simplified.

The position of the coordinate system is selected centered for the wrist exerciser which is also the TCP of the robot. The situation is represented on Fig. 3.

For cos and the sin the abbreviations c and s are used. Transformations are given with:

$${}^{2}\mathbf{T}_3(q_3) = Rot(z,q_3) = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$${}^{1}\mathbf{T}_2(q_2) = Rot(y,q_2) = \begin{bmatrix} cq_2 & 0 & sq_2 \\ 0 & 1 & 0 \\ -sq_2 & 0 & cq_2 \end{bmatrix}, \tag{6}$$

$${}^{0}\mathbf{T}_1(q_1) = Rot(z,\ q_1) = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

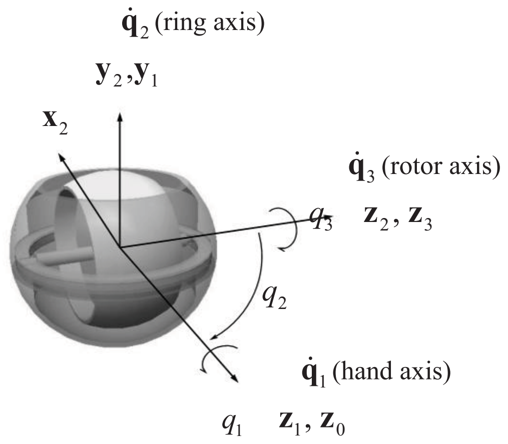

Figure 3. Coordinate systems, axis and rotation velocity vectors

Let the coordinate system CS 0 be chosen as inertial and the transformation (4) results in

$$ {}^{0}\mathbf{M} = \frac{d}{dt}\,{}^{0}\mathbf{\Gamma} = \frac{d}{dt}\left({}^{0}\mathbf{T}_2 \cdot {}^{2}\mathbf{\Gamma}\right), \tag{8} $$

$$ {}^{2}\mathbf{M} = {}^{2}\mathbf{T}_0\,{}^{0}\mathbf{M} = {}^{2}\mathbf{T}_0 \frac{d}{dt}\left({}^{0}\mathbf{T}_2\right){}^{2}\mathbf{\Gamma} + \frac{d}{dt}\left({}^{2}\mathbf{\Gamma}\right), \tag{9} $$

where are ${}^{0}\mathbf{T}_2 = {}^{0}\mathbf{T}_1\,{}^{1}\mathbf{T}_2$ and ${}^{2}\mathbf{T}_0 = \left({}^{0}\mathbf{T}_2\right)^{-1}$.

Express the transformation derivative with the tensor calculus:

$$ \frac{d}{dt}\,{}^{0}\mathbf{T}_2 = \mathbf{R}\left({}^{0}\mathbf{\Omega}\right)\cdot {}^{0}\mathbf{T}_2, \tag{10} $$

where is ${}^{0}\mathbf{\Omega}$ a rotation velocity vector of CS 2 in the CS 0.

Then from equation (9):

$$ {}^{2}\mathbf{M} = {}^{2}\mathbf{T}_0\mathbf{R}\left({}^{0}\mathbf{\Omega}\right){}^{0}\mathbf{T}_2\,{}^{2}\mathbf{\Gamma} + {}^{2}\dot{\mathbf{\Gamma}}, \tag{11} $$

and

$$ {}^{2}\mathbf{T}_0 \cdot \mathbf{R}\left({}^{0}\mathbf{\Omega}\right){}^{0}\mathbf{T}_2 = \mathbf{R}\left({}^{2}\mathbf{\Omega}\right), \tag{12} $$

torque in the CS of the ring is expressed with:

$$ {}^{2}\mathbf{M} = \mathbf{R}\left({}^{2}\mathbf{\Omega}\right){}^{2}\mathbf{\Gamma} + {}^{2}\dot{\mathbf{\Gamma}}. \tag{13} $$

The expression (13) is known as transport theorem:

$$ {}^{2}\mathbf{M} = {}^{2}\mathbf{\Omega} \times {}^{2}\mathbf{\Gamma} + {}^{2}\dot{\mathbf{\Gamma}}. \tag{14} $$

Where are quantities valid for CS 2 and are defined as:

$$ {}^{2}\mathbf{M} = \left[{}^{2}M_x, {}^{2}M_y, {}^{2}M_z\right]^T, \tag{15} $$

$$ \mathbf{R}\left({}^{2}\mathbf{\Omega}\right) = \begin{bmatrix} 0 & -{}^{2}\Omega_z & {}^{2}\Omega_y \\ {}^{2}\Omega_z & 0 & -{}^{2}\Omega_x \\ -{}^{2}\Omega_y & {}^{2}\Omega_x & 0 \end{bmatrix}, \tag{16} $$

$$ {}^{2}\mathbf{\Omega} = \left[\Omega_x, \Omega_y, \Omega_z,\right]^T = \dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_2 = \dot{q}_1\,{}^{2}\mathbf{z}_1 + \dot{q}_2\,{}^{2}\mathbf{y}_2, \tag{17} $$

$$^{2}\mathbf{\Gamma} = {}^{2}\mathbf{J} \cdot {}^{2}\boldsymbol{\omega}\,, \tag{18}$$

$$^{2}\mathbf{J} = {}^{3}\mathbf{J} = \begin{bmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J_3 \end{bmatrix}. \tag{19}$$

The entry J_3 is an principal angular momentum of the rotor spin axes and both J are angular momentums of the perpendicular principal axes to J_3. Equation (19) is thus valid for the rotor type of the body. Vector of the rotor rotation velocity is given in the component form:

$$^{2}\boldsymbol{\omega} = \dot{\mathbf{q}}_1 + \dot{\mathbf{q}}_2 + \dot{\mathbf{q}}_3 = \dot{q}_1\,{}^{2}\mathbf{z}_1 + \dot{q}_2\,{}^{2}\mathbf{y}_2 + \dot{q}_3\,{}^{2}\mathbf{z}_2\,. \tag{20}$$

When the matrix components are:

$$^{2}\mathbf{T}_1 = \left({}^{1}\mathbf{T}_2\right)^T = \left[{}^{2}\mathbf{x}_1, {}^{2}\mathbf{y}_1, {}^{2}\mathbf{z}_1\right] = \begin{bmatrix} cq_2 & 0 & -sq_2 \\ 0 & 1 & 0 \\ sq_2 & 0 & cq_2 \end{bmatrix}, \tag{21}$$

$$^{2}\mathbf{z}_1 = \begin{bmatrix} -sq_2 \\ 0 \\ cq_2 \end{bmatrix}, \quad {}^{2}\mathbf{z}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad {}^{2}\mathbf{y}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \tag{22}$$

then the velocities are given with:

$$^{2}\mathbf{\Omega} = \begin{bmatrix} -sq_2 \cdot \dot{q}_1 \\ \dot{q}_2 \\ cq_2 \cdot \dot{q}_1 \end{bmatrix} \quad \text{and} \quad {}^{2}\boldsymbol{\omega} = \begin{bmatrix} -sq_2 \cdot \dot{q}_1 \\ \dot{q}_2 \\ cq_2 \cdot \dot{q}_1 + \dot{q}_3 \end{bmatrix}. \tag{23}$$

When (15-23) are inserted in to the equation (13) the dynamical model of the free spinning rotor expressed in the ring CS is:

$$\begin{gathered} \begin{bmatrix} {}^{2}M_{\mathrm{x}} \\ {}^{2}M_{\mathrm{y}} \\ {}^{2}M_{\mathrm{z}} \end{bmatrix} = \begin{bmatrix} -Jsq_2 & 0 & 0 \\ 0 & J & 0 \\ J_3 cq_2 & 0 & J_3 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix}, \\ \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} -(2J - J_3)cq_2 \cdot \dot{q}_1\dot{q}_2 + J_3 \cdot \dot{q}_2\dot{q}_3 \\ (J_3 - J)sq_2 cq_2 \cdot \dot{q}_1^2 + J_3 sq_2 \cdot \dot{q}_1\dot{q}_3 \\ -J_3 sq_2 \cdot \dot{q}_1\dot{q}_2 \end{bmatrix}. \end{gathered} \tag{24}$$

The model (24) is important for expression of the nutation torque in equation (6) because the axis is parallel to $\mathbf{x}_2$:

$$\begin{aligned} M_N &= -{}^{2}M_x \\ &= Jsq_2 \cdot \ddot{q}_1 + (2J - J_3)cq_2 \cdot \dot{q}_1\dot{q}_2 - J_3 \cdot \dot{q}_2\dot{q}_3. \end{aligned} \tag{25}$$

The axis torques are the projections of the vector (15) on the axes of the system. The projection on $\mathbf{z}_3$ is trivial due to $\mathbf{z}_3 = \mathbf{z}_2$ and $M_3 = {}^{2}M_{\mathrm{z}}$. The same is valid for the ring $M_2 = {}^{2}M_{\mathrm{y}}$. In the wrist axis the transformation $M_1 = {}^{1}M_{\mathrm{z}} = \left({}^{1}\mathbf{T}_2\,{}^{2}\mathbf{M} \bullet {}^{1}\mathbf{z}_1\right) = -sq_2\,{}^{2}M_{\mathrm{x}} + cq_2\,{}^{2}M_{\mathrm{z}}$ is used, where (.) is a dot product.
Torques in the colocated rotations are given with:

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} H_{11} & 0 & H_{13} \\ 0 & H_{22} & 0 \\ H_{13} & 0 & H_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, \tag{26}$$

where are

$$H_{11} = Js^2 q_2 + J_3 c^2 q_2\,,$$

$$H_{13} = J_3 cq_2\,,$$

$$H_{22} = J\,,$$

$$H_{33} = J_3 ,$$
$$h_1 = -2(J_3 - J) sq_2 cq_2 \cdot \dot{q}_1 \dot{q}_2 - J_3 sq_2 \cdot \dot{q}_2 \dot{q}_3 ,$$
$$h_2 = h_y ,$$
$$h_3 = h_z .$$

For Powerball® activated only with the one wrist DOF torques M_2 and M_3 are zero. Coupling of the rotor with housing is modeled with the torque vector M_t :

$$\begin{bmatrix} M_1 \\ 0 \\ 0 \end{bmatrix} = H \cdot \ddot{q} + h(q, \dot{q}) + \begin{bmatrix} M_{t1} \\ M_{t2} \\ M_{t3} \end{bmatrix} . \tag{27}$$

The first component of this friction vector is again set to zero while this friction belongs to the actuated wrist axis. Remaining components are separated in dissipation and coupling terms:

$$\begin{bmatrix} M_{1t} \\ M_{2t} \\ M_{3t} \end{bmatrix} = \begin{bmatrix} 0 \\ M_{2d} + M_{2s} \\ M_{3d} + M_{3s} \end{bmatrix} . \tag{28}$$

First the dissipation effect is modeled. It has viscous ($\mathrm{k_V}$, $\mathrm{k_{V3}}$) and dry part ($\mathrm{k_{S3}}$) dependable from the state indicator ζ :

$$M_{2\mathrm{d}} = \mathrm{k_V} \cdot \dot{\mathrm{q}}_2 , \tag{29}$$

$$M_{3\mathrm{d}} = \begin{cases} \mathrm{k_{V3}} \dot{\mathrm{q}}_3 + \mathrm{k_{S3}} \cdot \mathrm{sign}\ (\dot{q}_3)\ ;\ \zeta = 0, \\ 0;\ \zeta \neq 0 \end{cases} . \tag{30}$$

Than the coupling friction is modeled, which depends on nutation torque M_N in the direction of x_2 and from slip velocity ν . Thru this port of the model the energy flows from the excitation in the rotor and back in direction to the wrist.

$$M_{2s} = -f_t\left(\nu, M_N\right) , \tag{31}$$

$$M_{3s} = -\zeta \cdot \frac{1}{p} \cdot f_t\left(\nu, M_N\right) \cdot \tag{32}$$

In the equations (31, 32) the function f_t represents dry friction (33) and is the constant k_S dry friction parameter acting between rotor axe and the grove of Powerball®.

$$f_t\left(\nu, M_N\right) = k_S sig\left(\nu\right) . \tag{33}$$

The rolling obeys relative velocity of the shaft and the precession in the housing:

$$\nu = \left|\zeta\right| \dot{q}_2 r_2 + \zeta \dot{q}_3 r_3 , \tag{34}$$

$$\zeta = \begin{cases} 1 & ;\ M_N > \mu \\ 0 & ;\ -\mu \le M_N \le \mu \\ -1 & ;\ M_N < -\mu \end{cases} .$$

Where ζ indicates the mode of the system. At ζ =1 the negative part of the rotors shaft is elevated and the positive side is down and at -1 is the situation opposite, and at 0 the rotor floats.

4 Parameter estimation

Developed Powerball® model serves for estimation of parameters k_S, k_V, k_{S3}, k_{V3} and torque μ with spin off experiment and simulation. The experiment trace and simulation results at different parameter sets are depicted on Fig. 4 (Time – spin)

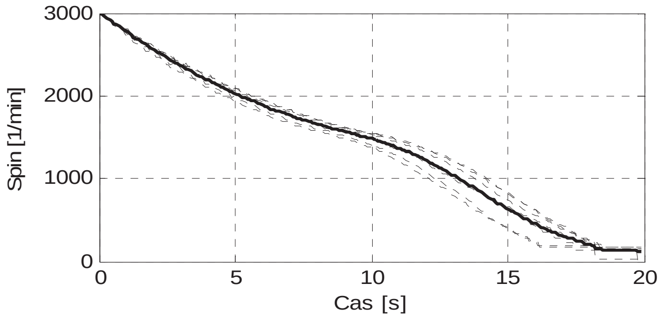

Figure 4.Experiment trace and simulation results of the spin-off

By chosing the functional F:

$$F = \sum_{k=1}^{n} \left| \dot{q}_3(k)_{(\exp.)}^{2} - \dot{q}_3(k)_{(sim.)}^{2} \right| , \tag{35}$$

which takes in account the only measured quantity $\dot{q}_3$ the estimated parameter set has optimized entities:
$k_S = 0.7468$, $k_V = 0.0068$, $k_{S3} = 0.0014$, $k_{V3} = 6.9830\text{e}-6$ and $\mu = 0.0148$.

The parameter set was than tested on the example which contains transition between two states (rolling-sliding) of Powerball® (Fig.5).

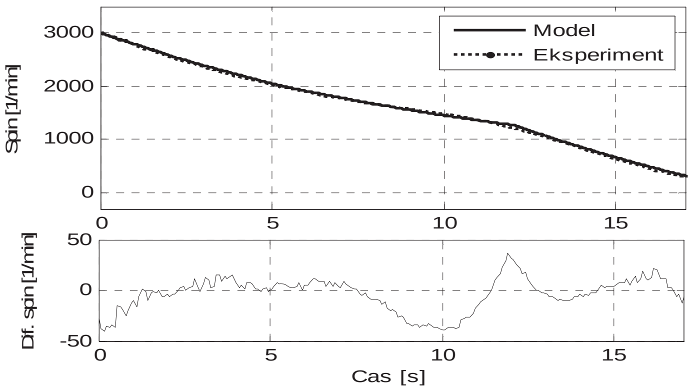

Figure 5. Rolling - sliding experiment and simulation results.

5 Control experiment

In the case of rolling the equation (3) enable to compute the excitation wrist torque components: first for coupling compensation , and second for the acceleration.

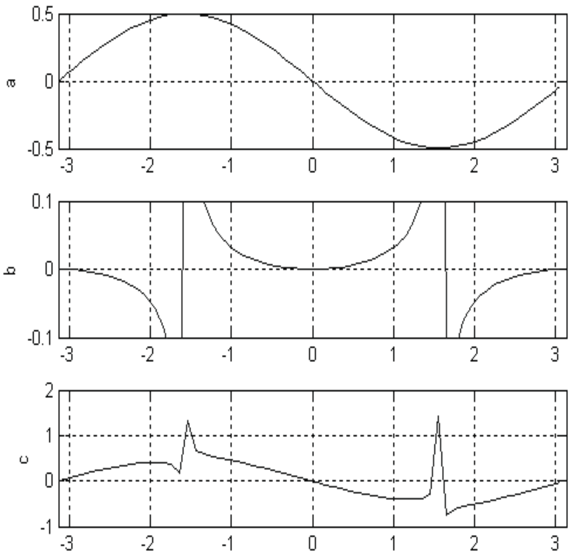

Figure 6. Accelerating in one rolling mode: a) torque for conditional rolling, b) torque component for acceleration, and c) resulting torque M_1.

Torque components and the resulting excitation are documented on Fig. (6). With acceleration torque it is easy to identify the nature of Powerball® coupling. For the one DOF excitation singular positions and the constant acceleration the effort becomes infinite.

Developed model serves also for calculating the cinematic variables:

$$\begin{aligned} \ddot{q}_2 &= \frac{1}{J}\left(T_{2S} - h_2(q,\dot{q}) - T_{2d}\right) \\ \ddot{q}_3 &= \frac{1}{J_3}\left(T_{3S} - h_3(q,\dot{q}) - J_3\cos(q_2)\ddot{q}_1 - T_{3d}\right) \end{aligned}, \qquad (36)$$

where is $\ddot{q}_1$, $\dot{q}_1$ and q_1 cinematic chain of wrist excitation for the Powerball® experiment shown on Fig. (7).

Experimental set-up is arranged around DSP2 controller and the servo drive Mini AX. Powerball® is clamped in an effector capable to measure spin speed only. The cinematic trajectory begins at 200 rad/s, than it is accelerated to 900 rad/s and on the end constant decelerated. The achievation results of spin are on Fig. (8).

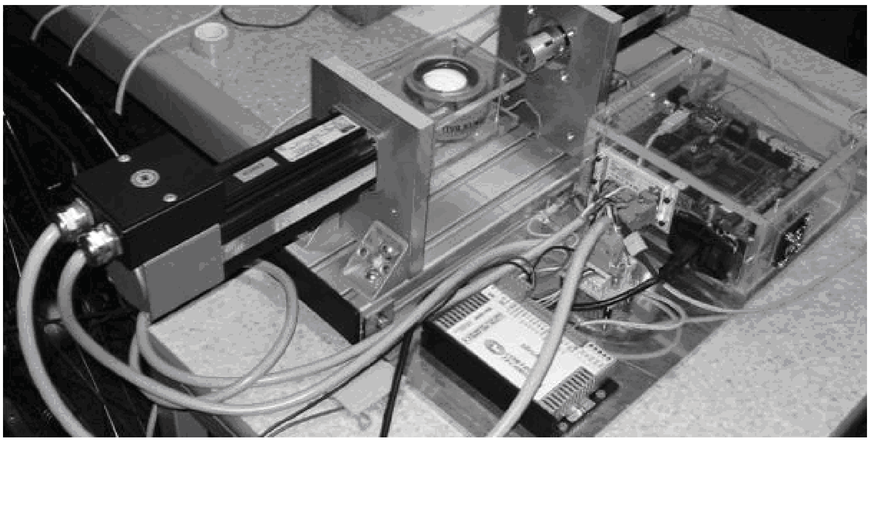

Figure 7. Directly coupled servo-drive and the Powerball®.

The rolling mode indicator ζ was either + or – 1 and was independent from the fact that the model is capable of reproducing rolling, transitions, and slip.

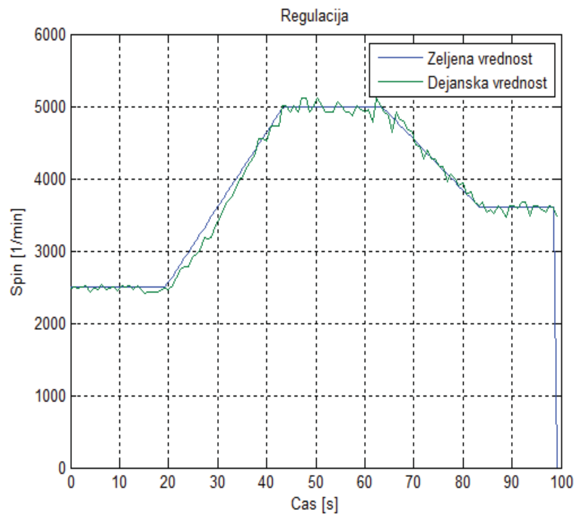

Figure 8. Spin kinematics control experiment.

6 Conclusions

The paper deals about modeling and experimenting with the gyroscopic device. First the model was developed for two DOF excitation and then reduced to one DOF suitable for our experiment setup. Simulation serves in model parameter estimation as well as for cinematic control development. The experiments show that the Powerball® can be accelerated and decelerated only with the output variable of rotor spin.

For the precession reversal the implicit torque signal should be considered. The research in this direction was performed with additional passive DOF and partly reported in [14].

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